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Installation and pricing decision in a two-channel supply chain under carbon cap-and-trade mechanism

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ABSTRACT

The rising global demand for energy has led to escalating issues related to resource scarcity, environmental pollution, and energy crises. In this study, we examine a two-channel supply chain operating under a carbon cap-and-trade mechanism, where manufacturers and retailers distribute low-carbon products that also come with installation services. We develop three models to analyze how service self-construction and outsourcing impact the decisions of different supply chain members. Additionally, we explore the effects of customer loyalty to retail channel, the difficulty of installation, and the prices in carbon trading on supply chain choices. Our findings indicate that customer loyalty to the retail channel influences manufacturers' and retailers' optimal installation service strategies. Notably, the model favoring the self-management of installation services is not the preferred choice for either manufacturers or retailers. Furthermore, we discover that excessively low and excessively high carbon trading prices hinder manufacturers from achieving optimal emission reductions.

1. Introduction

China has proposed to achieve carbon neutrality by 2060, advocating expanding the consumption and supply of low-carbon products(Xu et al. 2023).To address the challenge of carbon emissions, an increasing number of enterprises focus on the research, development, and sales of low-carbon products(Kong et al. 2023), including energy-efficient and household items(Ke et al. 2024). In this context, low-carbon products not only reflect consumers' environmental consciousness but also serve as a significant means to promote sustainable development and green consumption(Yang et al. 2023). However, the sales model for low-carbon products faces numerous challenges. Some consumers remain skeptical about these products' energy consumption and power generation capabilities, which hinders their widespread adoption worldwide. To combat this, we will explore retail strategies for low-carbon products, examining how to enhance market acceptance through effective marketing.

The choice of sales channels is critical to enterprises' market competitiveness and business development(Jia and Li 2020). Of these, the dual-channel model is the most prevalent. The dual-channel sales model maximizes the coverage of different types of customer groups and improves sales efficiency and market share. Gree, Midea, and other big brands have used the dual-channel model for years. In 2014, the photovoltaic giant JinkoSolar launched official flagship stores on Tmall, JD, Suning, and Gome, starting from the online and offline dual-channel sales model. However, introducing a dual-channel sales model also brings new challenges for enterprises. According to a previous study, as many as 49 percent of businesses in China go out of business due to improper sales models. Therefore, how to effectively implement the dual-channel sales model has become a problem that enterprises need to think deeply about and explore.

With the rapid development of the dual-channel supply chain, the economic position of manufacturers and retailers in the consumer market is gradually competing, and it is difficult to maintain the advantage by simply relying on product price competition. In incentivized price competition, enterprises can attract customers and maintain market share by providing value-added services(Zhang et al. 2025). For example, door-to-door delivery services for household products, warranty services for automobiles and electronic devices, etc.(Dai et al. 2025). Some energy and household low-carbon products need to be installed, and the installation effect will often affect the use of the products. For example, the installation effect of solar photovoltaic panels will affect the power generation of the photovoltaic system [1], and the installation effect of air conditioners will affect the heat dissipation and safety of air conditioners. Therefore, installation services are also seen as a bottleneck and key factor in the success of such products.

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This also reflects from the side that service planning is an important and challenging issue, especially in the product service system.(Moghadaspoor et al. 2024). We take the low-carbon products that need to be installed as the research object of this paper. Supply chain members face an important decision when choosing an installation service team: Do they build their installation team or outsource installation services? There are many factors involved in this decision, such as cost and quality assurance.

Building an installation team means that the company directly owns and manages the installation team, which gives better control over key factors such as installation quality and schedule, but at a higher cost. To build brand effect, some large manufacturers build their after-sales service teams and cooperate with some retailers to provide installation services to increase revenue [2], such as Siemens, Haier, etc. In contrast, some suppliers prefer to let the retailer provide the service [3]. For example, car manufacturers such as GM, Toyota, and Volkswagen have agreements with 4S retailers, who assume the responsibility of providing after-sales service. Outsourcing installation services, on the other hand, means delegating the installation tasks to other firms, which reduces the direct costs and administrative burden on the firms. However, it may also face the challenges of quality control and service competition. For example, Midea Enterprises will install the business to the retailer, provide its online purchase after the installation service, send the installation card to the manufacturer to apply, and send the manufacturer to the customer's return visit to confirm the settlement. In most photovoltaic companies, manufacturers and retailers mainly adopt the model of assuming installation services separately, such as Xikaide. There are many sales models among different enterprises, so exploring the most suitable sales model for the low-carbon products that need to be installed is essential. On this basis, we propose three decision models: (1) The manufacturer chooses to build its installation team while the retailer chooses to outsource its installation services to the manufacturer; (2) The retailer chooses to build its installation team while the manufacturer chooses to outsource its installation services to the retailer; (3) Both manufacturer and the retailer choose to build their installation teams. Outsourcing service costs are coordinated through the wholesale price [4].

As the market size of after-sales service continues to expand, the "service-product" bundling model has gradually become a strategy that attracts much attention [5]. Some industry service operators of smartphones and digital services bundle their core products and ancillary services for sale [6]. This strategy provides customers with a one-stop solution that can improve customer convenience and satisfaction. Lenovo, for example, sells personal computers and provides computer maintenance services to customers.

The supply chain members of the self-installed service team can adjust the service model at will. In contrast, the supply chain members of the service outsourcing will choose to label the product as a free service to obtain higher profits and long-term cooperation, which is actually to increase the sales price, that is, "service-product" bundle sales. Therefore, based on the three models, we assume that members who outsource services can only adopt the "service-product" bundle sales strategy to meet the model's applicability. On the contrary, members who choose to build their installation team can adopt the "service-product" bundle sales and only sell products.

It is worth noting that low-carbon products such as energy and home products need to be installed, and the installation effect often affects the stickiness of customers to the brand. In reality, some customers may require the product to be installed on specially shaped objects, or the installation location is relatively steep, which will significantly increase the difficulty of the construction. Installation work is often carried out according to customer requirements, installation environment, and installation standards, and professional installation services are required to calculate and propose installation plans and implement them. For example, for fixed-mount PV panels, a combination of angles maximizes the total annual output of PV panels [7]. Therefore, we assume that the more difficult it is to design the most suitable installation solution for the product, the more difficult the installation. In reality, most customers experience cost-effectiveness and will pay special attention to the installation effect. Therefore, we included the factor of installation difficulty in the study, which, together with the level of installation service provided by the merchant, affects the installation effect.

In this paper, we consider a two-channel supply chain under a carbon cap-and-trade scheme (cct), which involves a manufacturer and a retailer of low-carbon products that must be installed in two channels (direct sales and retail). For this kind of product, manufacturers and retailers must consider building their own or outsourcing a team of installation service professionals. The party that chooses to build its installation team can adopt a strategy of installation-service bundling or selling only products. The party that outsources the installation service can only adopt the "product-service" bundle sales model. To distinguish between these two sales models, we have developed a price discount that can meet the different types of needs of consumers. Through our research, we want to address the following questions: (1) What are the optimal installation strategy choices for manufacturers and retailers under a dual-channel structure? (2) How do customer loyalty to the retail channel, difficulty of installation, and carbon trading prices affect the supply chain under the cct mechanism? (3) What is the impact on the profitability of supply chain members if the manufacturer and retailer do not have an outsourcing partnership?

We derive equilibrium solutions for the three strategies of the manufacturer and the retailer. Research shows that when customer loyalty to the retail channel is small, the optimal strategy for the entire supply chain is for the manufacturer to outsource installation services to the retailer; when customer loyalty to the retail channel is large, the optimal strategy for the entire supply chain is for the retailer to outsource installation services to the manufacturer. In addition, increased installation difficulty incentivizes manufacturers and retailers to focus on product sales without investing too much in the installation service segment for profit. The results also suggest that a model in which manufacturers and retailers do not enter into installation service partnerships is not the first option for manufacturers and retailers.

At the same time, it can be seen that the innovation of this paper is:

(1) When manufacturers and retailers sell low-carbon products that need to be installed through dual channels, manufacturers and retailers can choose to build or outsource installation service teams according to their conditions, in which the cost of service outsourcing is coordinated through wholesale prices. On this basis, we establish three relevant models for analysis: Manufacturers choose to build their installation teams, while retailers choose to outsource installation services to manufacturers; Retailers choose to build their installation teams, while manufacturers choose to outsource installation services to retailers; Both manufacturers and retailers choose to build their installation teams.
(2) Based on the above three models, we add the bundled and separate sales strategies and set the discount price to explore the cooperative and competitive relationship between manufacturers and retailers.
(3) This paper's research object is the low-carbon products that need to be installed, and its sales strategy is discussed. Considering the professionalism of its installation operation, the installation difficulty parameter is set according to the installation environment, and the demand function is added. The installation effect is affected by the installation service level, and the installation difficulty's influence on the three model decision variables is explored.

The rest of the paper is structured as follows. Section 2 reviews the relevant literature. Section 3 introduces the model and briefly describes the notation used in this paper. Section 4 derives the equilibrium of manufacturers' and retailers' decisions in the three strategies. Section 5 analyzes the impact of customer loyalty to the retail channel and installation difficulty on manufacturer-retailer decision-making under the three strategies. Section 6 gives the management insight. Section 7 summarizes the article and suggests directions for future work.

2. Literature review

The literature on supply chains is organized into three subsections related to this study: Dual-channel in the supply chain, carbon cap-and-trade mechanism, and self-build and outsourcing model options. We will review these topics in the following subsections.

2.1. Dual-channel in the supply chain

The research in this paper is closely related to the direction of dual-channel structure research. Introducing a dual-channel structure involves several aspects, such as market coverage, competitive advantage, and supply chain synergy, which are crucial for enterprises to enhance their competitiveness and adapt to market changes. Rahmani & Yavari studied for the first time the management of demand disruptions in a green dual-channel supply chain that produces and sells green products [8]. On the other hand, Yu et al. investigated price and cold chain service decisions in a dual-channel fresh produce supply chain under retailer competition [9]. Similarly, Xie et al. studied a two-tier fresh food supply chain system consisting of a producer and a retailer. Based on this system, the authors investigated the optimal decision problem of carbon reduction and pricing under three two-channel supply chain sales models (retailer two-channel, producer two-channel, and hybrid two-channel) [10]. Also, dual-channel structures are particularly prevalent in the apparel industry. Xia et al. construct a two-period game model to examine the effects of social influences on channel sequence and pricing decisions in a dual-channel distribution system [11]. Many scholars have further studied the reverse recycling channel based on the dual channel. Zhang et al. developed a two-channel closed-loop supply chain model based on two types of defective product returns and scrap returns. Through the model, the scholars investigated how the closed-loop supply chain members determine the presence of defective product returns in terms of product quality and price [12]. Later, Moghadaspoor et al. recognized the significance of the leather industry to economic development and the environmental challenges it brought, and studied and designed a closed-loop leather supply chain network, including waste recycling and product regeneration processes(Moghadaspoor et al. 2024). Meanwhile, Chen et al. developed a model of a dual-channel closed-loop supply chain under a government penalty mechanism consisting of a dual-channel manufacturer, a retailer, and a government. The effects of the government RPM on the optimal decision of the supply chain system, the relationship between the two sales channels, and the total welfare of the society are analyzed [13]. The study of dual-channels is not limited to one type of supply chain, but also to multiple supply chain types that share a particular characteristic. Batarfi et al. considered the sale of customized products, where the manufacturer offers customized products in an online channel, focusing on the impact of adopting such dual channels on the performance of a two-tier supply chain [14]. In addition, Zhang et al. studied a retailer and a manufacturer who produce high-quality and low-quality products in a two-channel supply chain with product variability [15]. Yang et al. constructed a two-stage product-service supply chain consisting of a single manufacturing service provider and a single sales service integrator. It is thought of achieving data resource mining through cross-channel methods, thereby constructing an architecture of a dual-channel closed-loop supply chain. The higher the efficiency of data resource conversion between the two channels is, the higher the overall profit of the supply chain will be(Yang et al. 2024). The above literature studies different types of supply chains with dual-channel structures, mainly focusing on pricing strategies, channel conflicts, and coordination mechanisms. Previous studies have produced some important insights into dual-channel supply chains, but few scholars have studied bidirectional service selection among members of dual-channel supply chains. Based on the previous work, this paper designs a dual-channel model with installation service and bundled sales strategy and studies the pricing and service decisions in the supply chain.

2.2. Impact of carbon cap-and-trade mechanisms on low-carbon supply chains

Carbon cap-and-trade has attracted considerable attention from scholars in recent years as one of the important mechanisms to combat climate change and reduce greenhouse gas emissions. Pun & Ghamat explored a dual-channel setup involving manufacturers and retailers, where manufacturers, subject to cap-and-trade regulations, have undisclosed information on their carbon reduction costs. The research results show that high emission reduction costs can paradoxically benefit manufacturers, the environment, consumers, and the overall social welfare. In addition to ordinary products, some scholars have also studied low-carbon products under the carbon trading market(Pun and Ghamat 2025). Xia et al. developed a game model of low-carbon products and ordinary products to analyze the impact of carbon trading on low-carbon supply chains under different production modes. The results show that carbon trading can increase the unit retail price of ordinary products and low-carbon products [16]. Similarly, Zhang et al. explored the effects of total carbon regulate-and-trade policies, carbon allowances, and market competition intensity on the production choices of low-carbon products of competing supply chain manufacturers [17]. Similar to the study of low-carbon products is the selection of energy-saving strategies. Bai et al. investigated manufacturers' energy efficiency investments and their impact on endogenous supply chain structure under a cap-and-trade regime [18]. Ying et al. considered not only the forward sales channel but also the reverse channel of recycling and remanufacturing. The study focuses on changes in the expected utility of the supply chain and its members as a result of changes in four possibilities, namely, carbon trading price, consumer low-carbon awareness, carbon emissions, and competition from third-party recyclers, in three emission reduction models. The study shows that carbon trading price, consumer low-carbon awareness, and carbon emissions are negatively related to the expected utility of manufacturers and retailers [19]. Zou et al. also considered the risk aversion characteristics of retailers under the carbon quota trading system and constructed a network equilibrium model under the inventory capacity constraint based on the model to realize the low-carbon development of supply chain networks. The results show that the carbon trading mechanism can encourage enterprises to invest in carbon emission reduction and improve the low-carbon level of products [20]. To coordinate the supply chain, Xu et al. not only discussed the decision-making behaviors of the two-level sustainable supply chain under the cap-and-trade system but also investigated the coordination mechanism of the supply chain. The results reveal the influence of the trading price of unit carbon credits on the optimal decision variables in both centralized and decentralized systems [21] Unlike the above literature, Zhang et al. added factors of other carbon policies to the model and analyzed the impacts of a hybrid carbon policy combining carbon tax and cap-and-trade on manufacturers' carbon abatement activities and supply chain operations under the condition that the government imposes free and paid carbon emission allowances. The study shows that the carbon tax rate, carbon trading price, and quota payment ratio all have positive impacts on corporate emission reduction behavior, but negative impacts on corporate profits and social welfare [6]. Most of the existing literature has analyzed the impact of cap-and-trade on the supply chain under different circumstances, mainly focusing on emission reduction strategies and pricing decisions. It can be seen that the cap-and-trade system is important in various industries, so based on predecessors, we introduce the carbon cap-and-trade system into the dual-channel model we designed and focus on the impact of unit carbon trading price on the decision-making and profit of each member of the supply chain.

2.3. Self-build and outsourcing model options

In business operations, the choice between Self-built and outsourced services is an important strategic decision. This decision affects the direction of the enterprise's development and is closely related to its internal structure. Nowadays, many scholars have included how supply chain members or platforms can choose a more appropriate model in their research. Lou et al. discussed the outsourcing options of logistics services in a retailer-dominated supply chain where the retailer provides logistics services on its own or outsources them to a third-party logistics service provider. The results show that it is not always optimal for retailers to provide logistics services, although the double marginal effect is eliminated [22]. In contrast, Wang et al. investigated the optimal channel choice and logistics strategy for manufacturers. In the dual-channel structure, two delivery modes are considered for the manufacturer in its direct channel: a stand-alone logistics system or an e-retailer's logistics system [23]. Zhang et al. regarded the channel status as an important criterion for determining after-sales service outsourcing to understand the potential mechanisms driving the after-sales channel strategy (Zhang et al. 2025). To explore the impact of service outsourcing in supply chain power structures, Bian et al. considered three power structures: retailer-Stackelberg supply chains, vertical-Nash supply chains, and retailer-Stackelberg supply chains [24]. In addition, Yu et al. developed a game model of a fresh produce supply chain consisting of a supplier, a retailer, and a third-party logistics provider. The focus was on the impact of the outsourcing model of cold chain services on supply chain decisions and profits [25]. Similarly, Xing et al. developed a game-theoretic model to explore the strategic sourcing of logistics services in the shipping supply chain. In this model, freight forwarders are faced with the decision of whether to use an in-house or outsourcing model for inland logistics services, and whether to purchase ocean transportation services through forward agreements or in the spot market. The study suggests that freight forwarders should use the in-house model for inland logistics services, even if the cost of the in-house model is higher than the expected outsourcing cost [26]. Then, Zhou et al. studied the e-commerce closed-loop supply chain network equilibrium problem considering the retailer's blockchain technology input costs and the government blockchain subsidy quota under the outsourced logistics distribution services strategy and the self-built logistics distribution services strategy. The results show that when the blockchain technology input cost is low, the self-built logistics distribution services strategy is the best choice for e-retailers; when the blockchain technology input cost is high, the outsourced logistics distribution services strategy is the best choice for e-retailers(Zhou et al. 2024). Unlike service outsourcing, Lu et al. explored the choice between self-management and outsourcing of supply chain governance models from the perspective of core firms. The results show that the optimal governance model for a supply chain depends on the characteristics of the supply chain [27]. In addition to managing outsourcing, the study also addressed technology outsourcing options. Shi et al. examined the impact of government subsidy policies on a two-tier agricultural supply chain in which farmers could self-invest in green technologies or outsource them to service providers [28]. In summary, they studied the choice of self-employment and outsourcing mode from the perspective of different supply chain members and supply chain structures, including not only the outsourcing choice of service. However, few scholars have studied the situation in which manufacturers and retailers must choose between self-service and outsourcing. Based on previous studies, this study takes the low-carbon products that need to be installed as the research object and takes installation services as value-added services. Manufacturers and retailers must choose whether to build their installation team or outsource the service to the upstream and downstream. We propose three kinds of decision models based on the bundling strategy. The influence of installation difficulty on supply chain members' decision-making and mode selection is discussed.

3. Problem description and model assumptions

In this study, we examine a two-channel supply chain consisting of retailers (r) and manufacturers (m). Under the cct mechanism, manufacturers and retailers sell low-carbon products that must installed on the market. We construct the Stackelberg game model where the manufacturer is the dominant player, and the retailer is the follower. In the direct channel, the manufacturer sells the "product-service" bundle directly to the customer. In the retail channel, the manufacturer distributes the product or "product-service" bundle to the retailer, who then sells the "product-service" bundle to the customer. In addition, we consider three different installation service strategies in the dual channel: (1) Manufacturers offer installation services in both the direct and retail channels, and manufacturers can offer price discounts in the direct channel to customers who do not require installation services. In the retail channel, the manufacturer wholesales the "product-service" bundle to the retailer, who in turn sells the "product-service" bundle to the end customer (model M). (2) In the direct channel, manufacturers sell "product-service" bundles to consumers. In contrast, in the retail channel, manufacturers wholesale their products to retailers at a lower price to offset the cost of the service, and retailers can offer a price discount if the customer does not need the installation service. (model R). (3) Manufacturers and retailers offer installation services in the direct and retail channels, respectively, while manufacturers and retailers can each offer price discounts to end customers who do not require installation services (model MR). As Fig. 1 illustrates. Coordinate the prices of outsourcing services through wholesale prices.

In order to facilitate the model calculation, the main parameters and descriptions of the model built in this paper are shown in Table 1.

The following assumptions were made in this study to avoid trivial situations and ensure the manageability of the model developed.

Assumption 1. Taking into account the costs incurred by the manufacturer to produce each unit of product and the cost of goods sold incurred by the retailer to sell one unit of product, and for the sake of non-generality [29], we assume $c_m = c_r = 0$.

Assumption 2. The manufacturer has sufficient capacity to meet all demand for PV products from both channels and has no lost sales or backlog of orders [30].

Assumption 3. Different channels have different discounted prices, based on the actual situation, we assume that the discounts in the retail channel are greater than those in the direct sales channel i.e.:$\frac{1}{2} < \alpha_2 < \alpha_1 < 1$, and both α_1 and α_2 are exogenous variables.

Assumption 4. Demand for a product is linearly related to the price of the product and the level of installation service provided. When there are two channels in the model selling the same form of product (product only or "product-service" bundle), there is price competition between the two channels. When there are price discounts within the same channel, there is price competition within the channel. So we assume that the demand in the direct and retail channels is divided into two parts:$D_d = D_{dy} + D_{dn}$ and $D_r = D_{ry} + D_{rn}$.

Assumption 5. Carbon allowances can be bought and sold through the carbon trading market when the total amount of carbon emissions generated by a manufacturer producing low-carbon products need to be installed is greater or less than the total amount of carbon allowances. We can obtain the equation for total carbon emissions: $E = e(D_d + D_r)$, and G is the total amount of carbon emissions allocated by the

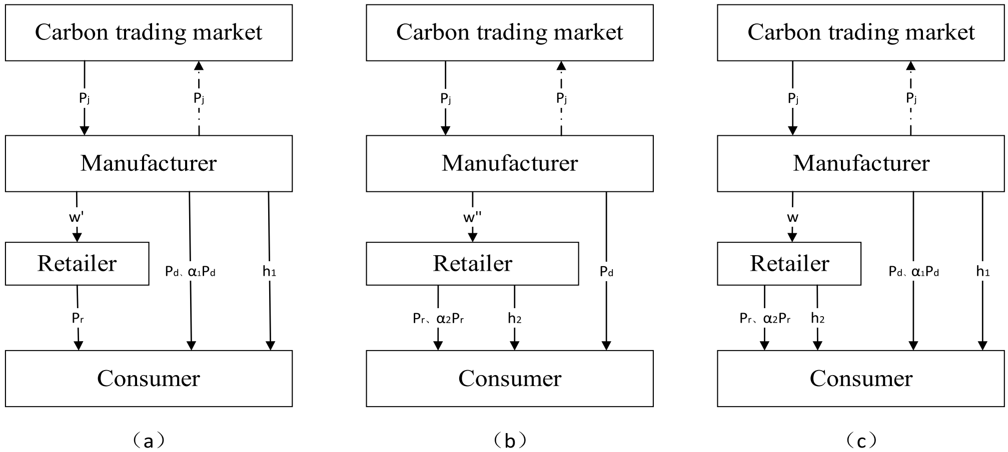

Fig. 1. Three installation service models.

Table 1
Description and interpretation of model symbols.

Symbols	Description and Explanation of Symbol
a	Potential market demand
θ	Customer loyalty to retail channel
t	The sensitivity of demand function to the selling price of products
α_1,α_2	Price discounts offered by manufacturer and retailer respectively
b_1	Cross-price sensitivity coefficient between different channels
b_2	Price sensitivity coefficient for products and "product-service" bundles within the same channel
u_1,u_2	Consumer preference for installation services in direct and retail channels.
k_1,k_2	Coefficient of influence of final service installation level on direct sales channel and retail channel
z	Investment cost coefficient for installation services
c	Unit production cost of the product ($/unit)
r	Sensitivity coefficient for installation difficulty
τ	Installation difficulty
e	Carbon emissions per unit of product (kg)
p_j	The trading price per carbon credit ($/kg)
G	Total carbon allowances received from the Government
w	Wholesale price when manufacturer and retailer services do not cooperate ($/unit)
w'	Wholesale price when the manufacturer offers a product-service bundle to the retailer ($/unit)
w''	The converted wholesale price of the product at which the retailer provides services to the manufacturer ($/unit)
p_d,p_r	Pricing of "product-service" bundles in direct and retail channels ($/unit)
f_1,f_2	Levels of installation services provided by manufacturer and retailer
D_{dy},D_{dn}	Demand for "product-service" bundles and demand for products only without installation in the direct channel
D_{ry},D_{rn}	Demand for "product-service" bundles and product-only demand without installation in the retail channel
π_m,π_r,π_{sc}	Representing manufacturer's profit, retailer's profit, and total supply chain profit, respectively
m,r,mr	Superscripts represent different modes

government. When $G - E < 0$, it indicates that the manufacturer needs to buy carbon credits from the carbon trading market to fulfill the carbon emission requirement. When $G - E > 0$, it indicates that the manufacturer has extra carbon emission credits that can be sold for extra profit [31].

Assumption 6. Regarding the investment cost of installation services and the level of installation services is not a linear relationship, the higher the level, the cost of services invested will rise sharply, so it is assumed that the investment cost function of the installation services is quadratic:$c(f_1) = zf_1^2/2$ and $c(f_2) = zf_2^2/2$, where z is the investment cost coefficient of the level of installation services [32], f_1 and f_2 are the level of installation services invested in by manufacturers and retailers, respectively.

Assumption 7. Because the installation of some low-carbon products needs to take into account the different requirements of the construction environment and customers in the region, the installation difficulty of the installation task is also different. We assume that installation difficulty is measured externally and assume that it is in the interval (0, 1). When the installation difficulty is low, it has less impact on the final presented installation service level. When the installation difficulty is high, it has more impact on the final presented installation service level. So a quadratic function is used to represent its impact on the installation service level: $h_i = f_i(1 - r\tau^2)$,where h_i indicates the final installation result, τ denotes the installation difficulty and r denotes the installation difficulty sensitivity factor. To ensure the validity of the function, we assume $\tau^2 < 1/r$.

Assumption 7. we assume that the market potential a is large enough such that the market demand is always positive.

4. Model building

In this section, we consider three dual-channel sales strategies with different installation service strategies where the manufacturer is the Stackelberg leader and the retailer is the follower. We use the Stackelberg game to build the corresponding decision model and solve it by the inverse solution method to obtain a closed-form solution with optimal decision variables. The leader in each model decides to maximize profits conditional on the response of the followers. The superscript "*" is used to identify the equilibrium solution.

4.1. Manufacturer-provided installation service mode (Model M)

In model M, the manufacturer partners with the retailer, with the manufacturer providing installation services in both the direct sales channel and the retail channel. In the direct channel, the manufacturer sells the “product-service” bundles to consumers at price p_d, but at discounted price $\alpha_1 p_d$ if the consumer does not need the installation service. In the retail channel, the manufacturer sells a "product-service" bundle to the retailer at price w', and the retailers sell the "product-service" bundle to consumers at price p_r, but without the discounted price if the consumer does not need the installation service. As shown in Fig. 1(a). The decision sequence for model M is that the manufacturer first determines the installation service level, wholesale price, and direct channel price, and then the retailer determines the retail price based on the wholesale price.

Based on the above description, the direct marketing channel requirements for model M are formulated as follows:

$$D_{dy}^{m} = a(1-\theta)u_1 - tp_d + b_1 p_r + b_2\alpha_1 p_d + k_1 h_1 \quad (1)$$

$$D_{dn}^{m} = a(1-\theta)(1-u_1) - t\alpha_1 p_d + b_2 p_d \quad (2)$$

Where D_{dy}^{m} denotes the demand function of consumers in the direct marketing channel who need to install the service and D_{dn}^{m} denotes the demand function of consumers in the direct marketing channel who do not need to install the service.

The retail channel requirements for model M are expressed below:

$$D_r^m = a\theta - tp_r + b_1 p_d + k_2 h_1 \quad (3)$$

The profits of the manufacturer, retailer and supply chain are as follows, respectively.

$$\pi_m^m = (w' - c)D_r^m + (p_d - c)D_{dy}^m + (\alpha_1 p_d - c)D_{dn}^m + (G-E)p_j - zf_1^2\Big/2 \quad (4)$$

$$\pi_r^m = (p_r - w')D_r^m \quad (5)$$

$$\pi_{sc}^m = (p_r - c)D_r^m + (p_d - c)D_{dy}^m + (\alpha_1 p_d - c)D_{dn}^m + (G-E)p_j - zf_1^2\Big/2 \quad (6)$$

In Eq. (4), the first term is the profit earned from wholesaling the "product-service" bundle through the retail channel, the second and third terms are the profit earned from selling the "product only" and "product-service" bundles through the direct sales channel, the fourth item is the benefits or costs incurred through carbon markets, and the fifth term is the cost of the investment in the installation service.

Proposition 1. *The profit of the manufacturer is jointly concave in w'^{m}, p_d^m, f_1^m provided that $b_2 < min\left\{\frac{t^2(\alpha_1^2+1)}{2\alpha_1 t}, \frac{\Omega_1+\Omega_2}{2\alpha_1(4tz-k_2^2(1-r\tau^2)^2)}\right\}$. By calculation we get the only feasible solution:*

$$p_d^{m*} = \frac{N_1 + N_2 + (N_3+N_4)N_7}{N_6 - N_5}$$

$$w'^{m*} = N_3 + N_4 + \frac{N_8(N_1+N_2+(N_3+N_4)N_7)}{N_6 - N_5}$$

$$f_1^{m*} = N_{11}(N_3+N_4) + \frac{N_{10}(N_1+N_2+(N_3+N_4)N_7)}{N_6-N_5} - N_9$$

$$p_r^{m*} = \frac{(N_{12} + 2zN_{11}N_{10})(N_1+N_2+N_{14}N_7) + (a\theta + tN_{14} + 2zN_{11}(N_{13}-N_9))(N_6-N_5)}{zt(N_6-N_5)}$$

Where $N_1 = a(1-\theta)(\alpha_1(1-u_1)+u_1) + \frac{a\theta b_1}{2t} - (c+ep_j)\left(b_1^2 + (b_2 - t)(1+\alpha_1) + b_1 + \frac{b_1(b_1-t)}{2t}\right)$, $N_2 = \left((1-r\tau^2)\left(k_1 + \frac{b_1 k_2}{2t}\right)\right)\left(-\left(\frac{(k_2(b_1-t-1)+k_1)(1-r\tau^2)}{z}\right)(c+ep_j)\right)$, $N_3 = \frac{2z(a\theta+(t-b_1)(c+ep_j))-4k_1k_2(1-r\tau^2)(c+ep_j)}{(4tz-k_2^2(1-r\tau^2)^2(2-t))}$, $N_4 = \frac{k_2^2(1-r\tau^2)^2(c+ep_j)(t-b_1-2)}{(4tz-k_2^2(1-r\tau^2)^2(2-t))}$, $N_5 = \frac{(1-r\tau^2)^2(b_1k_2+2tk_1)}{2t}\left(\frac{(b_1k_2+2tk_1)}{2zt} + \frac{k_2 t(4zb_1+b_1k_2^2(1-r\tau^2)^2+2k_1k_2(1-r\tau^2))}{2zt(4tz-k_2^2(1-r\tau^2)^2(2-t))}\right)$, $N_6 = -4\alpha_1 b_2 + 2t(\alpha_1^2-1) - \frac{b_1^2}{t} - \frac{b_1(4zb_1+b_1k_2^2(1-r\tau^2)^2+2k_1k_2(1-r\tau^2))}{2zt(4tz-k_2^2(1-r\tau^2)^2(2-t))}$, $N_7 = \frac{k_2(1-r\tau^2)^3(b_1k_2+2tk_1)}{4zt} + b_1$, $N_8 = \frac{(4zb_1+b_1k_2^2(1-r\tau^2)^2+2k_1k_2(1-r\tau^2))}{2zt(4tz-k_2^2(1-r\tau^2)^2(2-t))}$, $N_9 = (c+ep_j)\left(\frac{(1-r\tau^2)}{z}\left(k_2\left(1+\frac{b_1}{2t}-t\right)-k_1\right)\right)$, $N_{10} = \frac{k_2(1-r\tau^2)(4zb_1+b_1k_2^2(1-r\tau^2)^2+2k_1k_2(1-r\tau^2))}{4z^2t(4tz-k_2^2(1-r\tau^2)^2(2-t))} + \frac{(1-r\tau^2)}{z}\left(\frac{b_1k_2}{2t}+k_1\right)$, $N_{11} = \frac{k_2(1-r\tau^2)}{2z}$, $N_{12} = b_1 + \frac{t(4zb_1+b_1k_2^2(1-r\tau^2)^2+2k_1k_2(1-r\tau^2))}{2zt(4tz-k_2^2(1-r\tau^2)^2(2-t))}$, $N_{13} = \left(\frac{k_2^2(1-r\tau^2)^2(c+ep_j)(t-b_1-2)}{(4tz-k_2^2(1-r\tau^2)^2(2-t))} + \frac{2z(a\theta+(t-b_1)(c+ep_j))-4k_1k_2(1-r\tau^2)(c+ep_j)}{(4tz-k_2^2(1-r\tau^2)^2(2-t))}\right)\frac{k_2(1-r\tau^2)}{2z}$, $N_{14} = \frac{k_2^2(1-r\tau^2)^2(c+ep_j)(t-b_1-2)}{(4tz-k_2^2(1-r\tau^2)^2(2-t))} + \frac{2z(a\theta+(t-b_1)(c+ep_j))-4k_1k_2(1-r\tau^2)(c+ep_j)}{(4tz-k_2^2(1-r\tau^2)^2(2-t))}$.

Proofs of Proposition 1 and subsequent propositions are given in the Appendix.

4.2. Retailer-provided installation service mode (Model R)

In model R, the manufacturer enters into a partnership with the retailer and negotiates for the retailer to provide installation services in both the direct and retail channels. Within the direct channel, the manufacturer sells "product-service" bundles to consumers at price p_d, but does not receive a discounted price if the consumer does not require installation services. In the retail channel, the manufacturer wholesales the product at a lower converted price w'' to the retailer who needs to provide installation services to the direct channel, then the retailer sells the "product-service" bundle to the consumer at price p_r but at discounted price $\alpha_2 p_d$ if the consumer does not need installation services. The decision sequence for model R is: the manufacturer decides on the direct price, then the retailer decides on the level of installation service, the manufacturer sets the wholesale price based on the level of installation service provided by the retailer, and the retailer decides on the retail price. As shown in Fig. 1(b).

Based on the above description, the direct channel requirements for model R are formulated as follows:

$$D_d^r = a(1-\theta) - tp_d + b_1 p_r + k_1 h_2 \quad (7)$$

The retail channel requirements for model R are expressed below:

$$D_{ry}^r = a\theta u_2 - tp_r + b_1 p_d + b_2\alpha_2 p_r + k_2 h_2 \quad (8)$$

$$D_{rn}^r = a\theta(1-u_2) - t\alpha_2 p_r + b_2 p_r \quad (9)$$

Where D_{ry}^r denotes the demand function for consumers in the retail channel who require installation services, and D_{rn}^r denotes the demand function for consumers in the retail channel who do not require installation services.

The profits of the manufacturer, retailer, and supply chain are as follows, respectively.

$$\pi_m^r = (w'' - c)D_r^r + (p_d - c)D_d^r + (G-E)P_j \quad (10)$$

$$\pi_r^r = (p_r - w'')D_{ry}^r + (\alpha_2 p_r - w'')D_{rn}^r - zf_2^2\Big/2 \quad (11)$$

$$\pi_{sc}^r = (p_r - c)D_{ry}^r + (\alpha_2 p_r - c)D_{rn}^r + (p_d - c)D_d^r + (G-E)P_j - zf_2^2\Big/2 \quad (12)$$

In Eq. (10), the first term is the profit earned by wholesaling the product to the retail channel, the second term is the profit earned by selling the "product-service" bundle through the direct sales channel, and the third term is the benefits or costs incurred through carbon markets. In Eq. (11), the first term is the profit earned from selling the "product-service" bundle through the retail channel, the second term is the profit earned from selling the product through the retail channel, and the last term is the investment cost of installation service.

Proposition 2. *When $b_2 < min\left\{\frac{(\alpha_1^2+1)-b_1^2-\alpha_1 b_1}{\alpha_1}, \frac{\Omega_1+\Omega_2}{\alpha_1(4tz-k_2^2(1-r\tau^2)^2)}\right\}$, the retailer's profit is jointly concave in p_r^r and f_2^r. When $\Phi_1+\Phi_2<0$ and $4(\Phi_1+\Phi_2)\Phi_7 - (b_1\Phi_3+\Phi_4)(b_1\Phi_5+\Phi_6) > 0$, the manufacturer's profit is jointly concave in w''^r and p_d^r. The unique optimal solution is given by the following equation:*

$$p_d^r* = \frac{M_3M_8 + M_2(M_4+M_5+M_6+M_7)}{M_3M_9 - M_9(M_1+M_2)M_8}$$

$$w''^{r*} = \frac{(M_1+M_2)(M_3M_8+M_2(M_4+M_5+M_6+M_7))}{(M_4+M_5+M_6+M_7)(M_3M_9-M_2(M_1+M_2))}$$

$$p_r^r* = \left(\begin{array}{c}\left(M_{11}+\frac{M_1+M_2}{M_3}\right)\left(\frac{(M_1+M_2)(M_3M_8+M_2(M_4+M_5+M_6+M_7))}{(M_4+M_5+M_6+M_7)(M_3M_9-M_2(M_1+M_2))}\right)\\ +M_{12}\left(\frac{M_4+M_5+M_6+M_7}{M_3}\right)+M_{10}\end{array}\right)$$

$$f_2^r* = \frac{k_2(1-r\tau^2)}{z}\left(\begin{array}{c}\left(\frac{(M_1+M_2)(M_3M_8+M_2(M_4+M_5+M_6+M_7))}{(M_4+M_5+M_6+M_7)(M_3M_9-M_2(M_1+M_2))}\right) \\ \times\left(M_{11}+\frac{M_1+M_2}{M_3}-1\right)+M_{12}\left(\frac{M_4+M_5+M_6+M_7}{M_3}\right)+M_{10}\end{array}\right)$$

Where $M_1 = -b_1\left(\frac{k_2^2(1-r\tau^2)^2(2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2)}{z(\alpha_2+1)(t-b_2)-k_2^2(1-r\tau^2)^2}+1\right)$, $M_2 = \frac{k_1k_2(1-r\tau^2)^2}{z}\left(-1+\frac{z(\alpha_2+1)(t-b_2)-k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}\right)-\frac{-k_2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}$, $M_3 = \left(\frac{k_2^2(1-r\tau^2)^2}{z}+(\alpha_2+1)(b_2-t)\times 2\right)\left(\frac{z(\alpha_2+1)(t-b_2)-k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}\right)-\frac{k_2^2(1-r\tau^2)^2}{z}$, $M_4 = a\theta+\frac{z(\alpha_2+1)(b_2-t)a\theta(u_2+\alpha_2(1-u_2))}{z(2-4b_2\alpha_2+2\alpha_2^2)-k_2^2(1-r\tau^2)^2}$, $M_5 = (c+ep_j)\frac{k_1k_2(1-r\tau^2)^2}{z}\left(\frac{z(\alpha_2+1)(t-b_2)-k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}-1\right)$, $M_6 = (-(c+ep_j)((\alpha_2+1)(b_2-t)+b_1))\frac{z(\alpha_2+1)(t-b_2)-k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}$, $M_7 = \frac{(2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2)}{z(z(\alpha_2+1)(t-b_2)-k_2^2(1-r\tau^2)^2)}(k_2^2a\theta(u_2+\alpha_2(1-u_2))(1-r\tau^2)^2)$, $M_8 = \frac{b_1a\theta z(u_2+\alpha_2(1-u_2))\left(1+\frac{k_1k_2(1-r\tau^2)^2}{b_1z}\right)}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}-(c+ep_j)(k_2b_1(1-r\tau^2)^2)(k_1+k_2+b_2-t)+a(1-\theta)+\frac{b_1az(-(c+ep_j)((\alpha_2+1)(b_2-t)+b_1))}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}$, $M_9 = 2t-\frac{2zb_1^2+2b_1k_1k_2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}$, $M_{10} = \frac{za\theta(u_2+\alpha_2(1-u_2))}{z(2-4b_2\alpha_2+2\alpha_2^2)-k_2^2(1-r\tau^2)^2}$, $M_{11} = \frac{b_1z}{z(2-4b_2\alpha_2+2\alpha_2^2)-k_2^2(1-r\tau^2)^2}$, $M_{12} = \frac{z(\alpha_2+1)(t-b_2)-k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2)-k_2^2(1-r\tau^2)^2}$.

4.3. *Separate installation service models for manufacturers and retailers (Model MR)*

In model MR, manufacturers and retailers do not cooperate, with manufacturers providing installation services in the direct channel and retailers providing installation services in the retail channel. In the direct channel, the manufacturer sells a "product-service" bundle to the consumer at price p_d. In the retail channel, the manufacturer sells the product to the retailer at price w, then the retailer sells the "product-service" bundle to the consumer at price p_r. Discounted prices are available in both channels, $\alpha_1 p_d$ and $\alpha_1 p_r$, if consumers do not need installation services. The decision-making sequence of model MR is that the manufacturer decides on the wholesale and direct selling price and the level of installation service, and then the retailer decides on the retail price and the level of installation service in the retail channel. As shown in Fig. 1(c).

Based on the above description, the direct marketing channel requirements for model MR are formulated as follows:

$$D_{dy}^{mr} = a(1-\theta)u_1 - tp_d + b_1p_r + b_2\alpha_1p_d + k_1h_1 \tag{13}$$

$$D_{dn}^{mr} = a(1-\theta)(1-u_1) - t\alpha_1p_d + b_1\alpha_2p_r + b_2p_d \tag{14}$$

WhereD_{dy}^{mr} denotes the demand function of consumers in the direct marketing channel who need to install the service and D_{dn}^{mr}denotes the demand function of consumers in the direct marketing channel who do not need to install the service.

The retail channel requirements for model R are expressed below:

$$D_{ry}^{mr} = a\theta u_2 - tp_r + b_1p_d + b_2\alpha_2p_r + k_2h_2 \tag{15}$$

$$D_{rn}^{mr} = a\theta(1-u_2) - t\alpha_2p_r + b_1\alpha_1p_d + b_2p_r \tag{16}$$

Where D_{ry}^{mr} denotes the demand function for consumers in the retail channel who require installation services, and D_{rn}^{mr} denotes the demand function for consumers in the retail channel who do not require installation services.

The profits of the manufacturer, retailer, and supply chain are as follows, respectively.

$$\pi_m^{mr} = (w-c)D_r^{mr} + (p_d-c)D_{dy}^{mr} + (\alpha_1p_d-c)D_{dn}^{mr} + (G-E)P_j - zf_1^2\Big/2 \tag{17}$$

$$\pi_r^{mr} = (p_r-w)D_{ry}^{mr} + (\alpha_2p_r-w)D_{rn}^{mr} - zf_2^2\Big/2 \tag{18}$$

$$\begin{aligned}\pi_{sc}^{mr} &= (p_r-c)D_{ry}^{mr} + (\alpha_2p_r-c)D_{rn}^{mr} + (p_d-c)D_{dy}^{mr} + (\alpha_1p_d-c)D_{dn}^{mr} \\ &\quad + (G-E)P_j - zf_1^2\Big/2 - zf_2^2\Big/2\end{aligned} \tag{19}$$

In Eq. (17), the first term is the profit earned from wholesaling the product through the retail channel. The second and third terms are the profit earned from selling the "product-service" bundle and the "product only" through the direct sales channel, the fourth term is the portion expended or recovered from conducting the transaction in the carbon trading market, and the last term is the investment in the installation service costs. In Eq. (18), the first two terms are the profits earned through the sale of "product-service" bundles and "product only" in the retail channel, and the last term is the investment cost of the installation service.

Proposition 3. *When $2\alpha_2b_2 - t(1+\alpha_2^2) < 0$ and $2z(2\alpha_2b_2 - t(1+\alpha_2^2)) + k_2^2(1-r\tau^2)^2 < 0$, the retailer's profit is jointly concave in p_r^{mr} and f_2^{mr}. When $\Gamma_1 > 0$, $4\Gamma_1\Gamma_5 + b_1^2\Gamma_4(\Gamma_2+\Gamma_3) < 0$, and $4z\Gamma_1\Gamma_5 + b_1^2\Gamma_4(\Gamma_2+\Gamma_3) + 2\Gamma_1k_1^2(1-r\tau^2)^2 < 0$, the manufacturer's profit is jointly concave in w^{mr}, p_d^{mr} and f_1^{mr}. The unique optimal solution is given by the following equation:*

$$p_d^{mr*} = \frac{O_2(O_2+O_3)+O_2O_4(O_5-O_6)}{O_2O_8-O_4O_7}$$

$$w^{mr}* = O_5 - O_6\frac{O_7(O_2(O_2+O_3)+O_2O_4(O_5-O_6))}{O_2O_8-O_4O_7}$$

$$f_1^{mr*} = \left(\frac{k_1(1-\tau r^2)}{z_1}\right)\left(\frac{O_2(O_2+O_3)+O_2O_4(O_5-O_6)}{O_2O_8-O_4O_7}-\left(c+ep_j\right)\right)$$

$$\begin{aligned}p_r^{mr*} &= O_9 - O_{11}(O_5-O_6) \\ &\quad + (O_{10}+O_7O_{11})\frac{(O_2(O_2+O_3)+O_2O_4(O_5-O_6))}{O_2O_8-O_4O_7}\end{aligned}$$

$$\begin{aligned}f_2^{mr*} = &\left(\frac{k_2(1-r\tau^2)}{z}\right)\Bigg(O_9-(O_{11}+1)(O_5-O_6) \\ &+\frac{(O_{10}+O_7(O_{11}-1))}{O_2O_8-O_4O_7}(O_2(O_2+O_3)+O_2O_4(O_5-O_6))\Bigg)\end{aligned}$$

Where$O_1 = \frac{2(z(b_2-t)(\alpha_2+1)+k_2^2(1-r\tau^2)^2)^2}{z(2z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2)}+\frac{k_2^2(1-r\tau^2)^2}{z}$, $O_2 = a\theta(u_1+\alpha_1(1-u_1))+\frac{za\theta b_1(u_2+\alpha_2(1-u_2))(1+\alpha_1\alpha_2)}{2z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}-(c+ep_j)\left(\frac{k^2(1-r\tau^2)}{z}+(\alpha_1+1)(b_1+b_2-t)\right)$, $O_3 = zb_1(1+\alpha_1\alpha_2)\frac{\left(-(c+ep_j)\left((b_2-t)(\alpha_2+1)+\frac{k^2(1-r\tau^2)}{z}+b_1(1+\alpha_1)\right)\right)}{2z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}$, $O_4 = \left((b_2-t)(\alpha_2+1)+\frac{k_2^2(1-r\tau^2)^2}{z}\right)\left(\frac{b_1z(1+\alpha_1\alpha_2)}{(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}\right)-b_1(1+\alpha_1\alpha_2)\frac{z(b_2-t)(\alpha_2+1)+k_2^2(1-r\tau^2)^2}{(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}$, $O_5 = \frac{za\theta(u_2+\alpha_2(1-u_2))\left((b_2-t)(\alpha_2+1)+\frac{k_2^2(1-r\tau^2)^2}{z}\right)}{2z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}-(c+ep_j)\left(b_1(1+\alpha_2)-\frac{k_2^2(1-r\tau^2)^2}{z}\right)$, $O_6 = \frac{z(b_2-t)(\alpha_2+1)+k_2^2(1-r\tau^2)^2}{z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}(-(c+ep_j)((b_2-t)(\alpha_2+1)+k_2^2(1-r\tau^2)^2))$, $O_7 = b_1(1+\alpha_1)-\frac{z(b_2-t)(\alpha_2+1)+k_2^2(1-r\tau^2)^2(b_2+\alpha_1\alpha_2)}{z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}+\frac{b_1z(1+\alpha_1\alpha_2)}{(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}\left((b_2-t)(\alpha_2+1)+\frac{k_2^2(1-r\tau^2)^2}{z}\right)$,$O_8 = -\frac{2b_1^2z(1+\alpha_1\alpha_2)^2}{z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}-4\alpha_1b_2+2t(1+\alpha_1^2)-\frac{k_1(1-r\tau^2)}{z}$, $O_9 = \frac{za\theta(u_2+\alpha_2(1-u_2))}{2z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}$, $O_{10} = \frac{zb_1(\alpha_1\alpha_2+1)}{2z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}$, $O_{11} = \frac{z(b_2-t)(\alpha_2+1)+k^2(1-r\tau^2)}{2z(t(1+\alpha_2^2)-2\alpha_2b_2)-k_2^2(1-r\tau^2)^2}$.

5. Data analysis and discussion

Among such products studied, solar photovoltaic (PV) panels are one of the most commonly used in the energy category, which not only converts sunlight into electricity for residential, commercial, industrial, and agricultural use but also reduces dependence on limited resources and protects the environment and ecosystems (Aarakit et al. 2021). With the increasing demand for photovoltaic products in the household market and small business enterprises (Li et al. 2023), some photovoltaic enterprises are also gradually entering online channels for retail and are no longer limited to power stations and distributed construction projects. Guocheng Energy is one of China's famous photovoltaic manufacturing enterprises, committed to the production and sales of solar photovoltaic products, the use of a dual-channel distribution model to meet the family market and small industrial and commercial retail needs, with its own installation and construction team. However, its installation services are not shared with retailers; retailers will provide their installation services. Therefore, we will refer to their data for the following research to explore which model suits the sales and promotion of similar products. This section conducts a numerical study to validate the model results. First, in three models, we analyze the effects of customer loyalty to the retail channel, installation difficulty, and carbon trading price on supply chain members' optimal decisions and system outcomes, respectively. Second, we analyzed the impact of different installation decisions on supply chain members' final decisions. Finally, we compare the profits of the three models and analyze the optimal profits under various scenarios. In the following numerical research, the parameter values are set by a photovoltaic enterprise and the existing numerical research, and we make reasonable adjustments to the obtained data according to the paper's assumptions. The basic parameter Settings are as follows: a=600 [33], c=100, α_1=0.78, α_2=0.7, k_1=1.5, k_2=1.2, b_1=0.2, b_2=0.25, r=1.5, u_1=0.5, u_2=0.55, z = 500, e =1.5 [31], p_j=5 [34], G=100 [34]. In addition, the superscripts "m", "r" and "mr" in the graphs denote model M, model R, and model MR, respectively. The results of the following graphical data are obtained using the Matlab software.

5.1. Impact of customer loyalty to retail channels on decision outcomes

This section analyzes the impact of customer loyalty to the retail channel on the pricing decision, the installation service decision, and the profit in the three models, as well as the impact of the three models on the decisions of each member of the supply chain under fluctuations in customer loyalty to the retail channel. Customer loyalty to the retail channel is closely related to sales in the dual channel. We explore what decisions manufacturers and retailers should make when the two fluctuate and provide decision-making references for each supply chain member. When θ varies in the range [0.4, 0.8] in steps of 0.01, set τ = 0.3 and p_j =5. The numerical simulation results are shown in Table 2.

5.1.1. Impact of customer loyalty to retail channels on service decision

As shown in Fig. 2, the level of installation service provided by the manufacturers in model M and model MR all decrease as customer loyalty to the retail channel increases, while it can be seen that the level of installation service in model MR decreases faster than that in model M. In contrast, the level of installation service provided by the retailers in model R and model MR all increase with customer loyalty to the retail channel, and the growth trajectories almost overlap. This is because the increase in customer loyalty to the retail channel leads to an increase in demand for the retail channel, which in turn leads to a larger share of the manufacturer's profit from the retail channel. In model M, the two channels have homogenized installation services, and the competition between the two focuses on price competition. Although lowering the

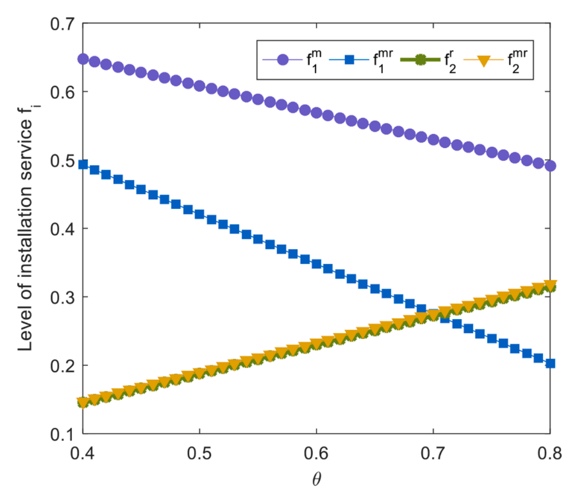

Fig. 2. Impact of customer loyalty to retail channel θ on installation service level f_i.

Table 2
Numerical simulation results of model M, model R, and model MR under the fluctuation of θ.

	θ	0.4	0.5	0.6	0.7	0.8
Model M	w' (\$/unit)	148.86	187.56	226.26	264.96	303.66
	p_d (\$/unit)	411.19	382.30	353.41	324.51	295.62
	p_r (\$/unit)	271.78	331.89	392.01	452.12	512.24
	f_1	0.64	0.60	0.56	0.53	0.49
	π_m (\$/unit)	28,944.31	26,349.57	26,373.05	29,014.73	34,274.61
	π_r (\$/unit)	2771.12	4978.39	7827.65	11,318.90	15,452.14
	π_{sc} (\$/unit)	31,715.48	31,327.96	342,000.70	40,333.63	49,726.75
Model R	w''(\$/unit)	129.66	157.41	185.16	212.90	240.65
	p_d(\$/unit)	455.33	416.64	377.96	339.27	300.58
	p_r(\$/unit)	229.61	277.75	325.89	374.03	422.17
	f_2	0.14	0.18	0.23	0.27	0.31
	π_m(\$/unit)	39,132.22	30,872.54	25,629.90	23,404.30	24,195.73
	π_r(\$/unit)	2269.58	3644.46	5437.30	7648.10	10,276.86
	π_{sc}(\$/unit)	41,401.8	34,517.00	31,067.2	31,052.4	34,372.59
Model MR	w(\$/unit)	166.02	172.55	199.08	225.61	252.14
	p_d(\$/unit)	417.49	389.52	361.56	333.59	305.63
	p_r(\$/unit)	246.89	294.06	351.22	388.38	435.55
	f_1	0.49	0.42	0.34	0.27	0.20
	f_2	0.14	0.19	0.23	0.23	0.31
	π_m(\$/unit)	29,024.56	24,831.12	22,814.74	22,975.40	25,313.11
	π_r(\$/unit)	1002.55	2380.92	4230.97	6552.70	9346.11
	π_{sc}(\$/unit)	30,027.11	27,212.04	27,045.71	29,528.1	34,659.22

level of installation service will reduce consumer stickiness in both channels, the direct sales channel has a price advantage. The price advantage of the retail channel and homogenized installation services makes some consumers return. Hence, the manufacturer prefers to reduce the level of installation service to recover losses in model M. In model MR, the manufacturer's installation services are not wholesaled to retailers, and the manufacturer cannot reap the profits from the service segment in the retail channel, so the manufacturer's installation service level in model MR declines faster than in model M. In model MR, there is no partnership between manufacturer and retailer. The reduced demand in the direct sales channel leads to decreasing profits for the manufacturer in the installation services segment, so the level of installation services the manufacturer provides declines more rapidly in model MR than in model M. At the same time, the increase in demand in the retail channel prompts retailers to raise the level of installation service to maintain their advantage and gain greater profits and customer stickiness. Therefore, in this scenario, manufacturers do not need to pay too much attention to the installation service segment. Manufacturers can set the installation service level at a reasonable range to recover the loss from the reduced demand in the direct channel. For retailers, it is more important to invest higher costs to improve the installation service level to increase consumer stickiness.

5.1.2. *Impact of customer loyalty to retail channels on pricing decisions*

For the price decision, as shown in Fig. 3 and 4, the wholesale and retail prices of the products in the three strategies increase as the customer's loyalty to the retail channel increases. In contrast, the direct price decreases as customer loyalty to the retail channel increases, as shown in Fig. 5. This result is intuitive. This is because increased customer loyalty to the retail channel leads to decreased demand for the direct channel. As a result, manufacturers are incentivized to lower prices in the direct sales channel to be more competitive. In this scenario, to mitigate the negative impact of reduced demand in the direct channel, the manufacturer should set a higher wholesale price to increase the marginal profit in the retail channel. Meanwhile, manufacturers can lower their direct selling prices for a more significant price advantage. Higher wholesale prices have also led manufacturers to set higher retail prices to maintain their equilibrium development.

The wholesale and retail prices of model M grow faster than the other two models, while model R grows at almost the same rate as model MR. This is because the decrease in the level of installation service in model M and the increase in the level of installation service in model R (as shown in Fig. 2) slows down the growth of wholesale and retail prices in their respective strategy. Moreover, the increase in the level of installed services in model R is greater than the decrease in the level of installed services in model M. Therefore, wholesale and retail prices increase

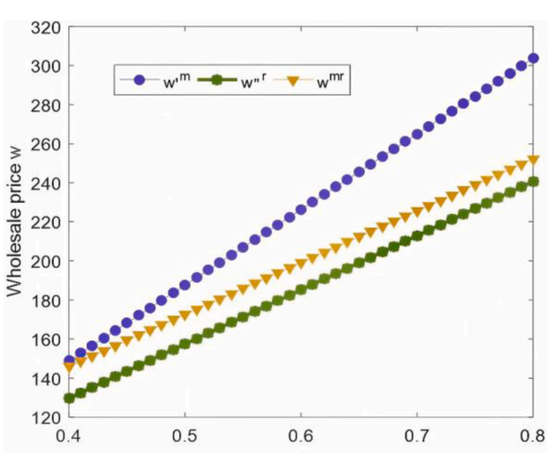

Fig. 3. Impact of customer loyalty to retail channels θ on wholesale prices w.

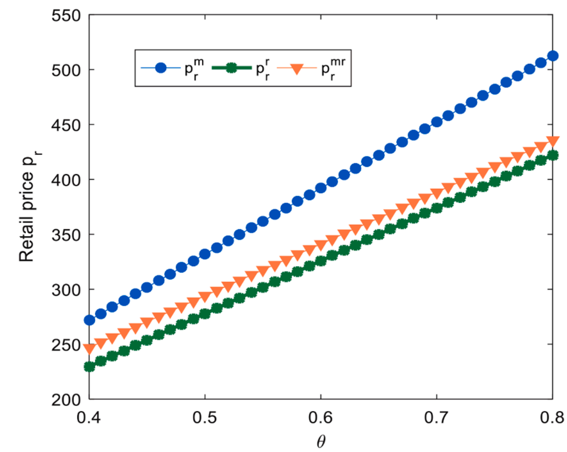

Fig. 4. Impact of customer loyalty to retail channels θ on retail prices p_r.

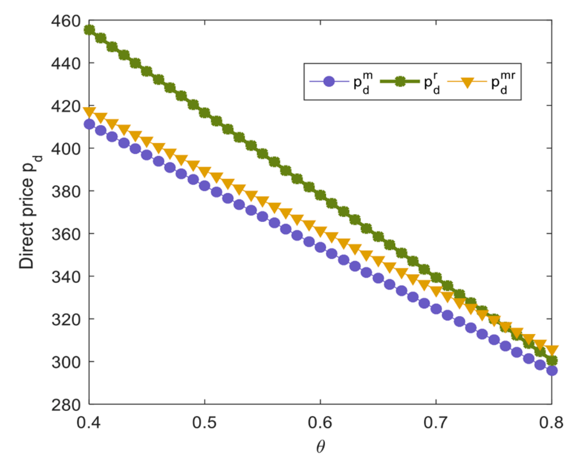

Fig. 5. Impact of customer Loyalty to retail channel θ on direct selling price p_d.

more rapidly in model M than in model R. In addition, because the increase in installed service levels in model R is greater than the decrease in installed service levels in model M, wholesale and retail prices increase faster in model M than in model R. Although installation services provided by the manufacturer do not directly affect wholesale price setting in model MR, an increase in the level of installation services provided by the retailer indirectly threatens the demand for installation services in direct channels so that wholesale and retail prices do not increase at a slower pace in model MR than in model R. As a result, manufacturers will increase wholesale prices more quickly under model M to escape the disadvantage caused by increased customer loyalty to the retail channel.

The model M has the highest wholesale and retail prices among the three models, the model MR is the next highest, and the model R is the smallest. In model M, the manufacturer wholesales the installation service to the retailer along with the merchandise. In contrast, in model R, the retailer provides the installation service to the customer in the direct sales channel. The manufacturer will set a relatively high wholesale price in model M and a lower wholesale price in model R to achieve installation service cooperation between the two, respectively. This also verifies the validity of the three models. In addition, the direct selling price of the goods is the highest in model R and the lowest in model M. This means that if the manufacturer sets a lower wholesale price in model R, it will lead to a reduction in the marginal profit that the manufacturer earns in the retail channel, in which case the manufacturer will choose to make up for the loss by setting a higher direct price in Model R.

In summary, supply chain members' pricing and installation service decisions interact with each other when changing customer loyalty to the retail channel. As customer loyalty to the retail channel increases, the competition between the manufacturer and the retailer becomes more intense. To cater to the advantages of increased customer loyalty to the retail channel, retailers may choose to increase the level of installation services to maintain customer stickiness. At the same time, retailers can increase their profits by raising retail prices. On the one hand, manufacturers should reduce direct prices to restore competitiveness in the direct channel. On the other hand, manufacturers need to reduce the level of installation services and increase wholesale prices to recover from the reduced demand in the direct channel.

5.1.3. Impact of customer loyalty to retail channel on profit

Fig. 6 shows the effect of customer loyalty to the retail channel on the manufacturer's profit, which decreases and then increases as customer loyalty to the retail channel increases in all three models. The transient decrease in the manufacturer's total profit is because as customer loyalty to the retail channel increases, the manufacturer's revenues in the direct channel decrease to a greater extent than they increase in the retail channel. The manufacturer then makes decisions, such as raising wholesale prices and lowering installation service levels, allowing the manufacturer's total profit to increase and eventually gradually become larger. It is very important to note that there exists a threshold for manufacturer profits as customer loyalty to the retail channel increases. When customer loyalty to the retail channel is below 0.58, the manufacturer's profits are greater in the R model; when customer loyalty to the retail channel θ exceeds this threshold, the manufacturer's profits are greater in the model M.

Fig. 7 represents the impact of customer loyalty to the retail channel on manufacturers' profits. Retailers' profits increase in response to increased loyalty to the retail channel. Due to year-on-year increases in retail and wholesale prices of products and increasing demand for the retail channel, making it profitable despite increased investment in installation services. This is mainly because although the product's retail price has increased in the same proportion as the wholesale price, the demand in the retail channel has continued to increase, so retailers remain profitable.

As the manufacturer's profit and the retailer's profit change, the total supply chain revenue decreases and then increases, as shown in Fig. 8. Furthermore, we find that the profit of the entire supply chain moves in the same direction as the manufacturer's profit with the same threshold. This is because not only do the retailer's profits move in the same direction across the three strategies, but the profits do not differ much at the same point.

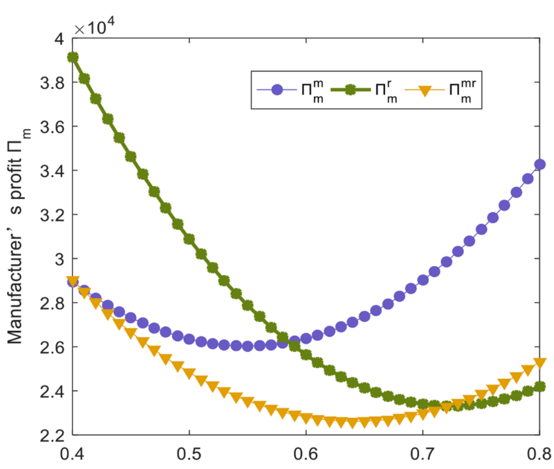

Fig. 6. Impact of customer loyalty to retail channel θ on manufacturer's profit π_m.

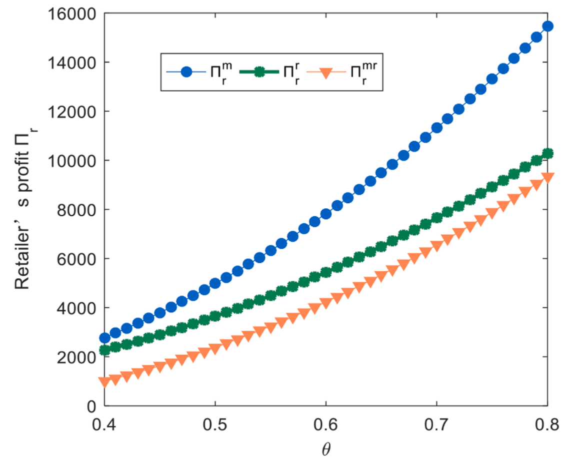

Fig. 7. Impact of customer loyalty to retail channels θ on retailers' profit π_r.

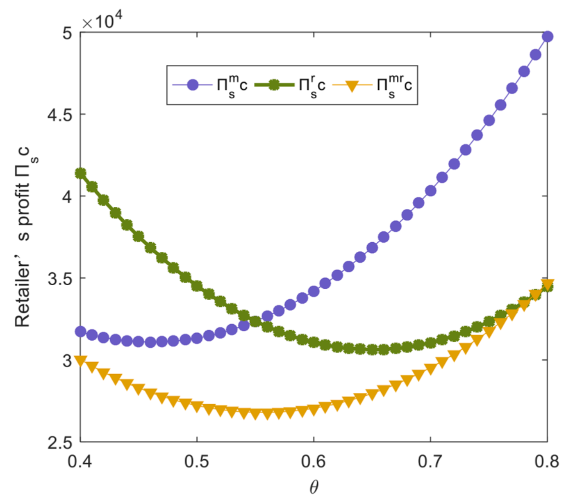

Fig. 8. Impact of customer loyalty to the retail channel θ on the profits of the whole supply chain π_{sc}.

Thus, as customer loyalty to the retail channel increases, the profits of the manufacturer and the entire supply chain fall and then rise while the profits of the retailer keep increasing. As θ approaches 1, the retailer's profits are maximized. In response, retailers can take specific measures to increase customer loyalty, such as targeted marketing, increasing promotions, and providing a good experience for customers who purchase products in brick-and-mortar stores. In addition, when θ is low, the manufacturer prefers to have the retailer provide installation services in both channels (model R). In contrast, the retailer prefers to have the manufacturer provide installation services in both channels (model M), so there is a conflict between manufacturers and retailers. To eliminate the conflict between manufacturers and retailers, one side must compromise, and the other needs to give a portion of its profits to compensate the compromised side. When θ is high, a "win-win" situation exists for both the manufacturer and the retailer if they choose to have the manufacturer provide installation services in both channels (model M).

5.2. Impact of installation difficulty on decision outcome

This section analyzes the impact of varying installation difficulty on pricing decisions, installation service decisions, and profitability under the three models. It also analyzes how different installation difficulty affects the choice of strategies by each supply chain member. Installation services provided by manufacturers and retailers face various challenges in carrying out their implementation, especially when it comes to the complexity of installation tasks, so it is essential to study the impact of installation difficulty on the entire supply chain. When τ varies in steps of 0.01 within the range [0.1, 0.8], let $\theta = 0.5$ and $p_j = 5$. The numerical simulation results are shown in Table 3.

5.2.1. Impact of installation difficulty on pricing and service decisions

As shown in Fig. 9. As the installation difficulty increases, the level of installation service provided by manufacturers and retailers gradually tends to zero. At the same time, the rate of decrease gradually accelerates. This implies that an increase in installation difficulty leads to increased customer attention to the installation service for a short time, followed by poorer installation feedback from the endpoints, which leads to customers no longer trusting the installation service in both channels. Lower trust leads to lower demand for installation services, thus resulting in less revenue for manufacturers and retailers in the installation service segment, which is why they tend not to offer installation services as the difficulty of installation increases.

Regarding pricing decisions, as shown in Fig. 10 and 11, the wholesale and retail prices of the products of model M and model MR decrease slightly as the installation difficulty increases. In model M, the manufacturer wholesales the installation service to the retailer along with the product. Hence, a decrease in the level of installation service triggers the manufacturer to set a lower wholesale price and the retailer to set a lower retail price. Whereas, in model R, a decrease in the level of installation services provided to the direct sales channel would lead to an increase in the wholesale price by the manufacturer. In model MR, greater installation difficulty leads to lower demand for the product. Hence, the manufacturer chooses to lower the product's wholesale price to attract retailers. In contrast, the lower level of installation service leads retailers to sell "product-install" bundles at reduced prices.

As shown in Fig. 12, the price of direct product sales decreases with increasing installation difficulty in all three models. This is because, in all three models, an increase in the difficulty of installation leads to a decrease in the level of installation service and, therefore, a decrease in the direct price of the "product-service" bundle.

As a result, as installation difficulty increases, all prices fall except the wholesale price of model R, which rises. The less complex the installation is, the higher the profit manufacturers and retailers receive for investing in installation services and the more incentive they have to provide better installation services. Additionally, we find that the manufacturer provides the highest level of installation service in model M.

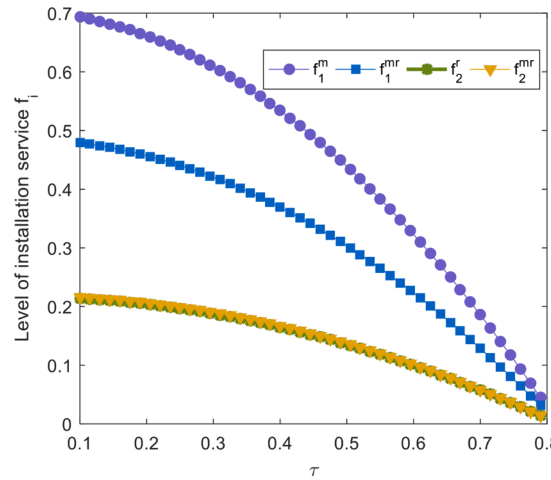

Fig. 9. Impact of installation difficulty τ on service installation levels f_i.

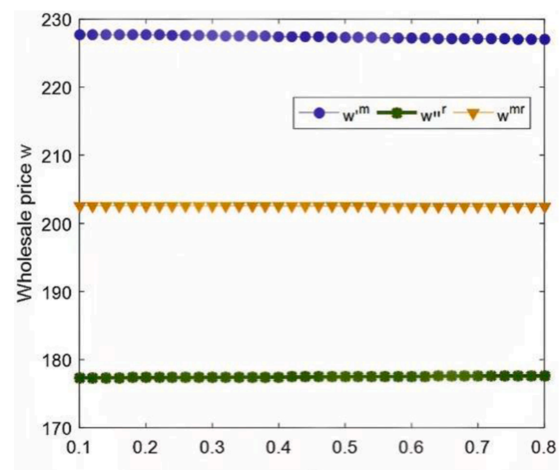

Fig. 10. Impact of Installation difficulty τ on wholesale prices w.

Table 3
Numerical simulation results of model M, model R, and model MR under the fluctuation of τ.

	τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Model M	w'($/unit)	227.72	227.66	227.56	227.44	227.31	227.18	227.08	227.03
	p_d($/unit)	257.46	257.40	257.30	257.17	257.03	256.9	256.8	256.75
	p_r($/unit)	382.12	382.03	381.89	381.72	381.53	381.35	381.21	381.14
	f_1	0.69	0.66	0.60	0.53	0.43	0.32	0.18	0.03
	π_m($/unit)	26,377.04	26,366.32	26,349.57	26,328.50	26,305.44	26,283.4	26,266	26,257.6
	π_r($/unit)	4986.33	4983.23	4978.39	4972.30	4965.65	4959.3	4954.29	4951.86
	π_{sc}($/unit)	31,363.37	31,349.55	31,327.96	31,300.8	31,271.09	31,242.7	31,220.29	31,209.46
Model R	w''($/unit)	167.35	167.37	167.41	167.45	167.50	166.50	167.58	167.60
	p_d($/unit)	296.7	296.68	296.64	296.60	296.55	296.50	296.46	296.44
	p_r($/unit)	327.76	327.76	327.75	327.75	327.74	327.73	327.73	327.72
	f_2	0.21	0.20	0.18	0.16	0.13	0.09	0.057	0.008
	π_m($/unit)	30,890.28	30,883.36	30,872.54	30,858.93	30,844.03	30,829.77	30,818.53	30,813.07
	π_r($/unit)	3650.62	3648.22	3644.46	3639.74	3634.57	3629.63	3625.73	3623.84
	π_{sc}($/unit)	34,549.9	34,531.58	34,517.00	34,498.67	34,478.6	34,459.4	34,444.26	34,436.92
Model MR	w($/unit)	182.57	182.56	182.55	182.53	182.513	182.49	182.47	182.46
	p_d($/unit)	269.64	269.59	269.52	269.44	269.34	269.26	269.18	269.15
	p_r($/unit)	344.12	344.09	344.06	344.01	343.95	343.90	343.86	343.84
	f_1	0.47	0.45	0.42	0.36	0.30	0.22	0.12	0.02
	f_2	0.21	0.20	0.19	0.16	0.13	0.10	0.05	0.008
	π_m($/unit)	24,848.91	24,841.97	24,831.12	24,817.47	24,802.52	24,788.22	2374.12	24,771.47
	π_r($/unit)	2383.16	24,841.97	2380.92	2379.21	2377.33	2375.54	24,776.94	2373.43
	π_{sc}($/unit)	27,232.07	27,224.26	27,212.05	27,196.68	27,179.86	27,163.76	27,151.07	27,144.91

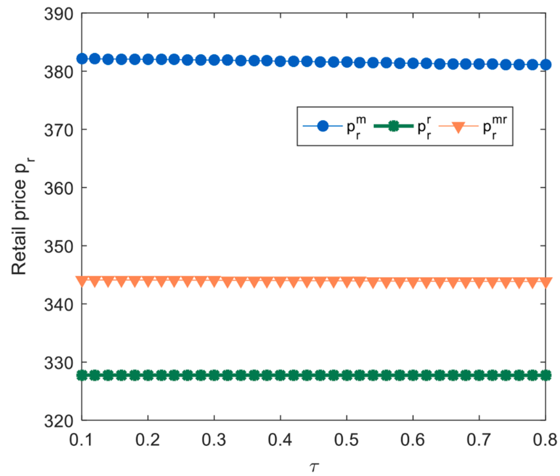

Fig. 11. Impact of installation difficulty τ on retail prices p_r.

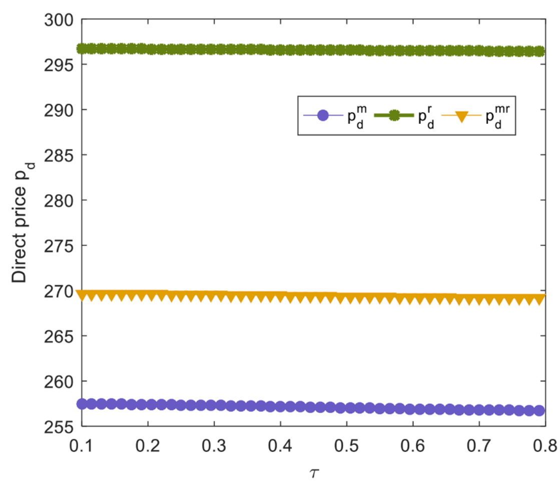

Fig. 12. Impact of installation difficulty τ on direct prices p_d.

5.2.2. Impact of installation difficulty on profit

As shown in Fig. 13–15, in all three strategies, the manufacturer's and retailer's profits decrease as installation difficulty increases. The main reason is that increased installation difficulty reduces the product's direct and retail price. It can also be seen that no matter how difficult the installation changes are, the manufacturer can maximize the profit in the model R. In contrast, the retailer can maximize the profit in the model M.

(Fig. 14)

With the change in the manufacturer's and retailer's profits, the total supply chain revenue decreases with the increase in installation difficulty, as shown in Fig. 15. In model R, the total return of the dual-channel supply chain is the largest.

Therefore, increasing installation difficulty will decrease profits for manufacturers and retailers. When installation difficulty is low, manufacturers and retailers can improve the quality and efficiency of installation services through training, technical support, and standardization of processes, thereby improving the user's consumption experience. In addition, retailers should try to focus their target groups on users who live in places where installation is less complex. If retailers choose to operate stores in areas with high installation difficulties, they can try to reduce the negative impact by negotiating with the government and applying for government subsidies. In scenarios where the difficulty of installation fluctuates, In the case of fluctuating installation difficulty, two members of the supply chain are more inclined to outsource services. This option will increase the profitability of both parties.

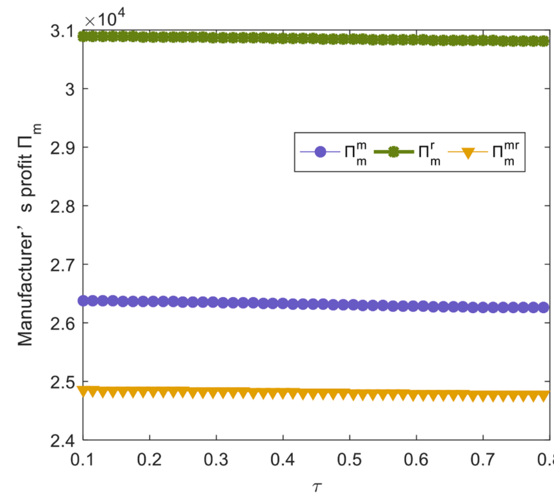

Fig. 13. Impact of Installation difficulty τ on manufacturers' profit π_m.

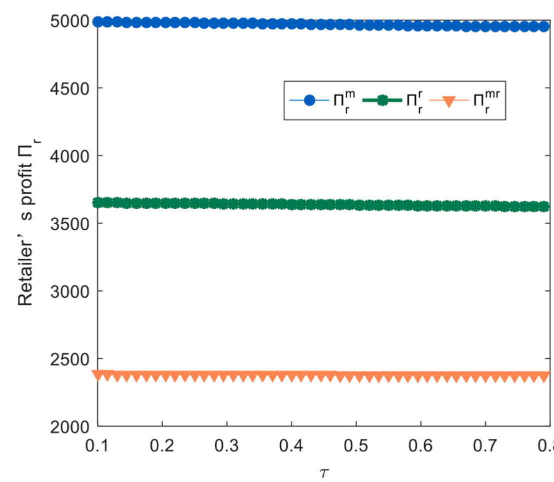

Fig. 14. Impact of Installation difficulty τ on retailers' profit π_r.

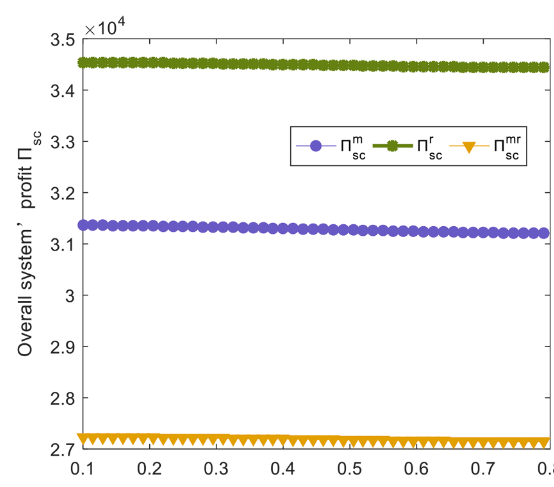

Fig. 15. Impact of Installation difficulty τ on the profits of the whole supply chainπ_{sc}.

5.3. Impact of carbon trading prices on decision outcomes

This section analyzes the impact of carbon trading prices on the decisions and profits of supply chain members in the three models. When $p_j$varies in steps of 1 in the range [0, 10], let $\theta = 0.5$ and $\tau = 0.3$. The numerical simulation results are shown in Table 4.

As shown in Fig. 16, the optimal level of service provided by manufacturers and retailers negatively correlates with the carbon trading price under the three strategies. Optimal service levels provided by manufacturers are more sensitive to carbon trading prices. This is mainly because manufacturers have direct dealings with the carbon market, so manufacturer's decisions are more influenced by the price of carbon trading. Higher carbon trading prices increase the cost of production for manufacturers, who do not want to see their total carbon emissions exceed the carbon limits set by the government. So manufacturers save money by lowering their service levels appropriately. At the same time, an appropriate reduction in the level of service by the manufacturer can prevent the market demand from increasing excessively.

As shown in Fig. 17–18, retail and direct prices positively correlate with carbon trading prices under the three strategies. This means that an increase in carbon trading price induces manufacturers to buy fewer emission rights or sell more, thus forcing them to make forced reductions. Manufacturers reduce their service levels, leading to reduced demand, which results in a loss of profit for both manufacturers and retailers. Manufacturers and retailers have had to recoup their losses by increasing their selling prices, but higher prices have led to further reductions in demand, so an increase in the price of carbon trading has prompted supply chains to tend to cut back on production. In this scenario, a higher carbon trading price, while conducive to creating an environment that reduces emissions, is detrimental to manufacturers and retailers, as shown in Fig. 19–20.

Therefore, for manufacturers companies not to be overly affected by the price of carbon trading, they can choose to optimize the production process to implement carbon reduction measures. Although the carbon market mainly determines the carbon trading price. The government is the regulator of carbon trading, so it is not appropriate for the government to set the carbon trading price too high when implementing the relevant policies. Otherwise, it will have the opposite effect and discourage enterprises from reducing emissions. A carbon trading price that is too low cannot effectively guide carbon emission reduction and may even increase carbon emissions. In this context, the government could set a range of carbon trading prices so that the prices would not be too low or too high.

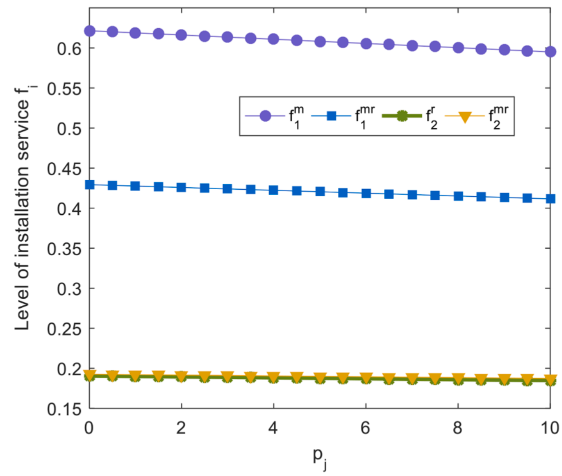

Fig. 16. Impact of carbon trading prices p_j on installation service levels f_i.

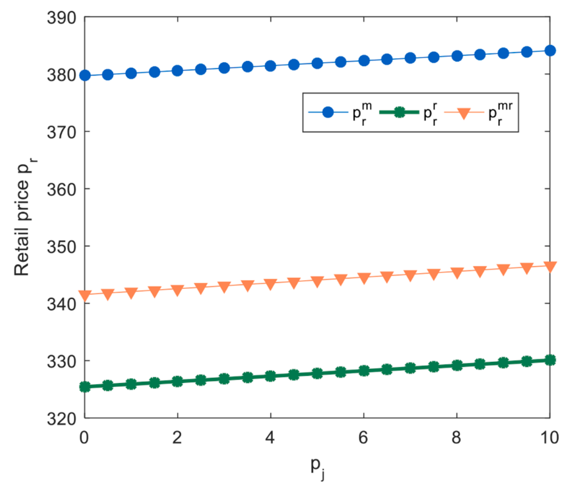

Fig. 17. Impact of carbon trading prices p_j on retail price p_r.

Table 4
Numerical simulation results of model M, model R, and model MR under the fluctuation of p_j.

	p_j($/kg)	0	2	4	6	8	10
Model M	w'($/unit)	243.77	245.29	246.80	248.32	249.83	251.35
	p_d($/unit)	253.20	254.84	256.48	258.12	259.75	261.39
	p_r($/unit)	379.72	380.59	381.46	382.33	383.20	384.07
	f_1	0.621	0.615	0.611	0.605	0.600	0.595
	π_m($/unit)	27,134.31	26,816.30	26,503.78	26,196.74	25,895.19	25,599.13
	π_r($/unit)	5170.53	5093.24	5016.53	4940.40	4864.85	4789.88
	π_{sc}($/unit)	32,304.85	31,909.54	31,520.31	31,137.14	30,760.04	30,389.01
Model R	w''($/unit)	183.69	185.18	186.66	188.15	189.63	191.12
	p_d($/unit)	292.87	294.38	295.89	297.40	298.91	300.42
	p_r($/unit)	325.44	326.37	327.29	328.22	329.14	330.07
	f_2	0.19	0.189	0.187	0.186	0.185	0.184
	π_m($/unit)	31,712.30	31,372.79	31,038.0	3613.32	30,383.11	30,062.84
	π_r($/unit)	3803.38	3739.17	3675.82	30,708.20	3551.69	3490.93
	π_{sc}($/unit)	35,515.68	35,111.96	34,713.91	34,321.53	33,934.81	33,553.77
Model MR	w($/unit)	198.78	200.29	201.79	203.30	204.81	206.32
	p_d($/unit)	265.44	267.08	268.71	270.34	271.97	273.60
	p_r($/unit)	341.54	342.55	343.55	344.56	345.56	346.57
	f_1	0.428	0.425	0.422	0.418	0.415	0.411
	f_2	0.192	0.191	0.190	0.189	0.188	0.187
	π_m($/unit)	25,660.05	25,324.32	24,994.13	24,669.50	24,350.42	24,036.90
	π_r($/unit)	2559.35	2487.48	2416.27	2345.74	2275.87	2206.66
	π_{sc}($/unit)	28,219.41	27,811.80	27,410.41	27,015.25	26,626.3	26,243.56

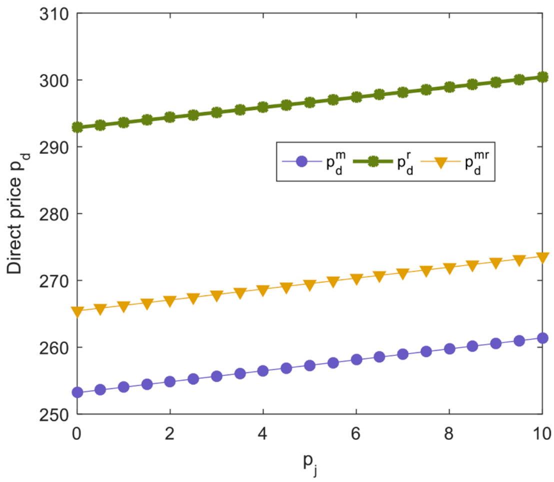

Fig. 18. Impact of carbon trading prices p_j on direct prices p_d.

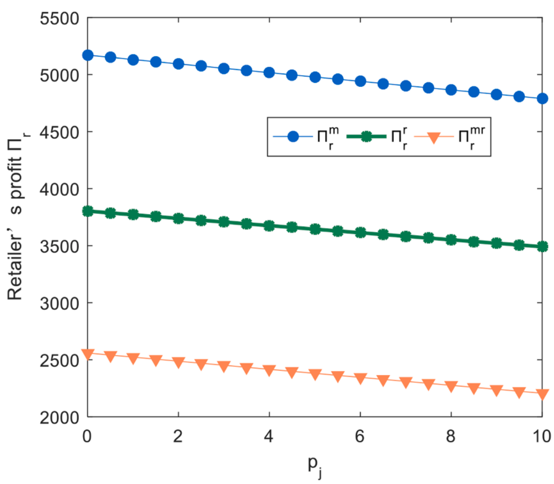

Fig. 19. Impact of carbon trading prices p_j on retailers' profit π_r.

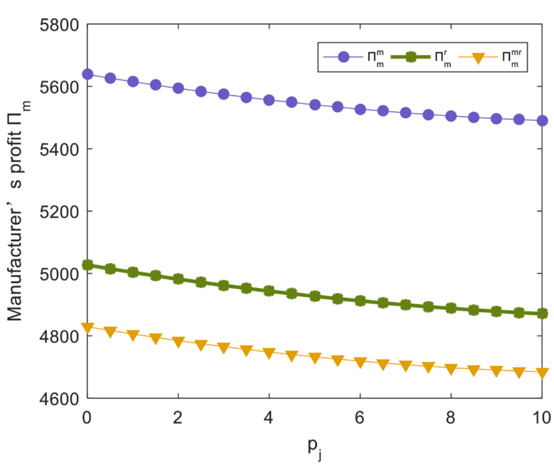

Fig. 20. Impact of carbon trading prices p_j on manufacturers' profit π_m.

6. Managerial insights

Based on the key findings, we offer the following valuable guidance for managers. Firstly, when latent demand or online loyalty is high, it can negatively impact the retailer. Retailers should promote their offerings and enhance service quality to improve customer retention. However, when profitability appears bleak, some retailers may opt to lower service levels as a cost-saving measure, this trade-off is often unavoidable. Retailers might consider adopting a low-price strategy, aiming for modest profits through high sales volumes, while partnering with large manufacturers who can provide installation services to stimulate demand in the offline market and mitigate unfavorable conditions. Conversely, if retailers have sufficient capacity, offering installation services can effectively counter the challenges posed by e-commerce. When manufacturers and retailers are more inclined to outsource these services, retailers can leverage negotiations with manufacturers to secure better terms and develop installation services to their advantage, which is crucial for long-term growth.

For manufacturers, the choice to refrain from collaboration is not ideal, as it leads to resource wastage and undermines the potential for a sustainable partnership with retailers. This paper posits that making informed decisions regarding installation services can enhance product value and generate additional business revenue. Large manufacturers have the opportunity to establish dedicated professional installation service teams to capitalize on the increasing demand in the offline marketplace.

In practice, an increasing number of companies, such as Gree and Midea, are competing in the after-sales service sector, intensifying supply chain competition. Given the varying proportions of online and offline demand for different products, manufacturers and retailers can conduct market research and cost analyses to determine whether to develop their professional service teams. Additionally, they can consider offering differentiated outsourcing services to boost revenue.

Furthermore, the installation complexity of various low-carbon products can differ significantly. For instance, the installation of energy-saving washing machines is less complex compared to that of photovoltaic solar panels and air conditioners. Therefore, different companies should judiciously form their professional service teams based on the specific needs associated with their products.

7. Conclusion

In this paper, we consider a two-channel supply chain under the cct mechanism, where the supply chain consists of a manufacturer and a retailer, both of which sell low-carbon products that must be installed in a dual channel. Based on this supply chain structure, three different hybrid models of installation services and bundling strategies are explored, and the impact of these three models on supply chain profitability is analyzed. Through numerical simulation, we investigate the effects of customer loyalty to the retail channel, installation difficulty, and carbon trading level on optimal pricing strategy, installation service strategy, and supply chain performance. Also, we compared the three installation service models under the variation of each of the three parameters to find the optimal model for all members.

Through numerical studies and theoretical analysis, we summarize our main findings as follows. First, we find that regardless of parameter fluctuations, the highest level of installation service is provided in model M. Consumers enjoy a better installation service experience. Second, the optimal model for manufacturers and retailers depends on customer loyalty to the retail channel. When customer loyalty to the retail channel is low, the manufacturer will prefer to work with the retailer to provide installation services in both channels. In contrast, the retailer prefers the manufacturer to provide installation services in both channels. For the two to agree, one must compromise, and the other will need to compensate the other with a portion of its profits. When customer loyalty to the retail price is high, a "win-win" situation can be achieved when the manufacturer and retailer choose to work with the manufacturer providing installation services. The model MR is unsuitable for most enterprises, and enterprises can consider cooperating with retailers according to their scale to obtain greater benefits. Third, the more difficult the installation in areas, manufacturers and retailers are less willing to provide high-quality installation services, which is inconsistent with the government's expectations for solving problems such as remote mountain power supply, so the relevant enterprises can choose to seek the help of the government or relevant departments to negotiate a solution. Finally, a carbon trading price that is too low or too high will not motivate producers to achieve optimal emission reduction, so the government should set a reasonable range of carbon trading prices when implementing the relevant policies.

The present work still has some limitations. First, this paper examines the dual-channel supply chain in which manufacturers and retailers exist, but the market is mainly competitive. Future studies may be conducted in more complex situations, such as having multiple competing manufacturers and retailers, multiple products, or including third-party service providers. In addition, to simplify the model, we assume that the outsourcing cooperation between the two is coordinated through the product's wholesale price. However, in practice, some enterprises sign contracts with outsourcing service providers and can charge service fees in various ways, such as the number of times. Therefore, multiple service cooperation models between supply chains can be expanded.

CRediT authorship contribution statement

Jinyu Wei: Writing – review & editing. **Yu Wang:** Writing – original draft. **Yaoxi Liu:** Writing – review & editing, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

Proof of Proposition 1

First, from the first-order condition of optimization$\frac{\partial \pi_r}{\partial p_r}=0$,since $\frac{\partial^2 \pi_r}{\partial {p_r}^2}=-2t<0$,the profit of the retailer is concave. The retailer sets the retail price of standard products as follow.

$$p_r^m*(w,p_d,f_1)=\frac{a\theta+b_1p_d+tw'+k_2(1-r\tau^2)f_1}{2t} \tag{A.1}$$

Take Equation (A.1) into Eq. (4), then the manufacturer maximizes his profit π_m and determines the optimal decisions. Take the second-order partial derivatives of π_m with respect to w', p_d and f_1 obtain Hessian Matrix.

$$H(w',p_d,f_1)=\begin{pmatrix} -t & b_1 & \frac{k_2(1-r\tau^2)}{2} \\ b_1 & 4\alpha_1b_2-2t(\alpha_1^2+1)-\frac{b_1^2}{t} & (1-r\tau^2)\left(k_1+\frac{b_1k_2}{2t}\right) \\ \frac{k_2(1-r\tau^2)}{2} & (1-r\tau^2)\left(k_1+\frac{b_1k_2}{2t}\right) & -z \end{pmatrix}$$

In order to prove the programmability of a Hessian matrix, the following equations need to be satisfied:

$$|H_{2\times2}|=t\left(t(\alpha_1^2+1)-2\alpha_1b_1\right)>0$$

$$|H_{3\times3}|=\left(4\alpha_1b_2-2t(\alpha_1^2+1)-\frac{b_1^2}{t}\right)\left(tz-\frac{k_2^2(1-r\tau^2)^2}{4}\right)+b_1^2z+\left(b_1k_2\left(k_1+\frac{b_1k_2}{2t}\right)+t\left(k_1+\frac{b_1k_2}{2t}\right)^2\right)(1-r\tau^2)^2<0$$

When $b_2<\frac{t^2(\alpha_1^2+1)}{2\alpha_1t}$, $|H_{2\times2}|>0$ holds. When $b_2<\frac{\Omega_1+\Omega_2}{\alpha_1(4tz-k_2^2(1-r\tau^2)^2)}$, $|H_{3\times3}|<0$ holds.

Therefore, the profit function of the manufacturer π_m is jointly concave in w', p_d and f_1 provided that $b_2<min\left\{\frac{(\alpha_1^2+1)-b_1^2-\alpha_1b_1}{\alpha_1},\frac{\Omega_1+\Omega_2}{\alpha_1(4tz-k_2^2(1-r\tau^2)^2)}\right\}$. where$\Omega_1=2t\left(\alpha_1^2+1+\frac{b_1^2}{t}\right)\left(tz-\frac{k_2^2(1-r\tau^2)}{4}\right)-b_1^2z$, $\Omega_2=-\left(b_1k_2\left(k_1+\frac{b_1k_2}{2t}\right)+t\left(k_1+\frac{b_1k_2}{2t}\right)^2\right)(1-r\tau^2)^2$. Mean-while, the unique optimal solution exists.

Bringing (1.1) into the manufacturer's profit function (4), The optimal solutions of w', p_d and f_1 given in Proposition 1 can be found by solving $\frac{\partial\pi_m}{\partial w'}=0$, $\frac{\partial\pi_m}{\partial p_d}=0$, $\frac{\partial\pi_m}{\partial f_1}=0$.

Proof of** Proposition 2.

For the concavity of the profit function given in Eq. (11), the Hessian matrix, which is given as follow, must be negative definite.

$$H(p_r,f_2)=\begin{pmatrix} 2\left(2b_2\alpha_2-t(\alpha_2^2+1)\right) & k_2(1-r\tau^2) \\ k_2(1-r\tau^2) & -z \end{pmatrix}$$

In order to prove the programmability of Hessian matrix, the following equations need to be satisfied:

$$|H_{1\times1}|=2\left(2b_2\alpha_2-t(\alpha_2^2+1)\right)<0$$

$$|H_{2\times2}|=-2z\left(2b_2\alpha_2-t(\alpha_2^2+1)\right)-k_2^2(1-r\tau^2)^2>0$$

When$b_2 \langle \frac{t(\alpha_2^2+1)}{2\alpha_2}, |H_{1\times1}| \rangle 0$holds.When $b_2 < \frac{2zt(\alpha_2^2+1)+k_2^2(1-r\tau^2)^2}{4z\alpha_2}, |H_{2\times2}| > 0$ holds. When these two conditions are satisfied, the profit function of retailer π_r is concave on p_r and f_2.

The optimal solutions of p_r and f_2 can be found by solving $\frac{\partial\pi_r}{\partial p_r} = 0$ and $\frac{\partial\pi_r}{\partial f_2} = 0$:

$$p_r^r * (w'', p_d) = \frac{a\theta z(u_2 + \alpha_2(1-u_2)) + b_1 z p_d + w''\left(z(t-b_2)(\alpha_2+1) - k_2^2(1-r\tau^2)^2\right)}{2z(1 - 2\alpha_2 b_2 + \alpha_2^2) - k_2^2(1-r\tau^2)^2} \tag{A.2}$$

$$f_2 * (w'', p_d) = \left(\frac{k_2(1-r\tau^2)}{z}\right)\left(\frac{a\theta z(u_2 + \alpha_2(1-u_2)) + b_1 z p_d}{2z(1 - 2\alpha_2 b_2 + \alpha_2^2) - k_2^2(1-r\tau^2)^2} + \frac{w''\left(z(t-b_2)(\alpha_2+1) - k_2^2(1-r\tau^2)^2\right)}{2z(1 - 2\alpha_2 b_2 + \alpha_2^2) - k_2^2(1-r\tau^2)^2} - w''\right) \tag{A.3}$$

After getting the reactions of the retailer, the manufacturer maximizes his profit and determines the optimal solutions. The Hessian matrix associated with the profit function π_m is given by

$$H(w'', p_d) = \begin{pmatrix} 2(\Phi_1 + \Phi_2) & b_1\Phi_3 + \Phi_4 \\ b_1\Phi_5 + \Phi_6 & 2\Phi_7 \end{pmatrix}$$

In order to prove the programmability of Hessian matrices, the following formulas need to be satisfied:

$$|H_{1\times1}| = \Phi_1 + \Phi_2 < 0$$

$$|H_{2\times2}| = 4(\Phi_1 + \Phi_2)\Phi_7 - (b_1\Phi_3 + \Phi_4)(b_1\Phi_5 + \Phi_6) > 0$$

Where $\Phi_1 = \frac{k_2^2(1-r\tau^2)^2}{z}\left(\frac{z(\alpha_2+1)(t-b_2) - k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2) - k_2^2(1-r\tau^2)^2} - 1\right)$,$\Phi_2 = 2(b_2 - t)(\alpha_2 + 1)\left(\frac{z(\alpha_2+1)(t-b_2) - k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2) - k_2^2(1-r\tau^2)^2}\right)$,$\Phi_3 = b_1\left(\frac{-k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2) - k_2^2(1-r\tau^2)^2}\right)$,$\Phi_4 = \left(\frac{k_1 z^2(\alpha_2+1)(1-b_2) - k_2^2(1-r\tau^2)^2 + b_1 k_2^3(1-r\tau^2)^2}{2k_2 z^2(1-2b_2\alpha_2+\alpha_2^2) - k_2^2(1-r\tau^2)^2} - \frac{k_1}{k_2} + b_1\right)$,$\Phi_5 = \left(\frac{-k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2) - k_2^2(1-r\tau^2)^2} + 1\right)$,$\Phi_6 = \frac{k_2(1-r\tau^2)^2}{k_1 z}\left(\frac{z(\alpha_2+1)(t-b_2) - k_2^2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2) - k_2^2(1-r\tau^2)^2} - 1\right)$,$\Phi_7 = \frac{b_1^2 z^2 + b_1 k_1 k_2(1-r\tau^2)^2}{2z(1-2b_2\alpha_2+\alpha_2^2) - k_2^2(1-r\tau^2)^2} - t$.

Therefore, the profit function of the manufacturer π_m is jointly concave inw'' and p_d provided that $\Phi_1 + \Phi_2 < 0$ and $4(\Phi_1 + \Phi_2)\Phi_7 - (b_1\Phi_3 + \Phi_4)(b_1\Phi_5 + \Phi_6) > 0$. Mean-while, the unique optimal solution exists.

Bringing (A.2)-(A.3) into the manufacturer's profit function (10), and let $\frac{\partial\pi_m}{\partial w''} = 0$ and $\frac{\partial\pi_m}{\partial p_d} = 0$.The optimal solutions of w'' and p_d given in Proposition 2 can be found.

Proof of Proposition 3

For the concavity of the profit function given in Eq. (18), the Hessian matrix, which is given as follow, must be negative definite.

$$H(p_r, f_2) = \begin{pmatrix} 2\left(2\alpha_2 b_2 - t\left(1+\alpha_2^2\right)\right) & k_2\left(1-r\tau^2\right) \\ k_2\left(1-r\tau^2\right) & -z \end{pmatrix}$$

In order to prove the programmability of Hessian matrices, the following formulas need to be satisfied:

$$|H_{1\times1}| = 2\left(2\alpha_2 b_2 - t\left(1+\alpha_2^2\right)\right) < 0$$

$$|H_{2\times2}| = -2z\left(2\alpha_2 b_2 - t\left(1+\alpha_2^2\right)\right) - k_2^2\left(1-r\tau^2\right)^2 > 0$$

When$2\alpha_2 b_2 - t(1+\alpha_2^2) < 0$, $|H_{1\times1}| > 0$holds.When$-2z(2\alpha_2 b_2 - t(1+\alpha_2^2)) - k_2^2(1-r\tau^2)^2$, $|H_{2\times2}| > 0$holds. When these two conditions are satisfied, the profit function of retailer π_r is concave on p_r and f_2.

The optimal solutions of p_r and f_2 can be found by solving $\frac{\partial\pi_r}{\partial p_r} = 0$ and $\frac{\partial\pi_r}{\partial f_2} = 0$:

$$p_r * (w, p_d) = -\frac{k_2^2(1-r\tau^2)^2 w}{2z(t(1+\alpha_2^2) - 2\alpha_2 b_2) - k_2^2(1-r\tau^2)^2} + \frac{z(b_1(1+\alpha_1\alpha_2)p_d + (1+\alpha_2)(t-b_2)w + a\theta(u_2 + \alpha_2(1-u_2)))}{2z(t(1+\alpha_2^2) - 2\alpha_2 b_2) - k_2^2(1-r\tau^2)^2} \tag{A.4}$$

$$f_2 * (w, p_d) = \frac{k_2(1-r\tau^2)}{z}\left(\frac{-k_2^2(1-r\tau^2)^2 w}{2z(t(1+\alpha_2^2) - 2\alpha_2 b_2) - k_2^2(1-r\tau^2)^2} - w + \frac{z(b_1(1+\alpha_1\alpha_2)p_d + (1+\alpha_2)(t-b_2)w + a\theta(u_2 + \alpha_2(1-u_2)))}{2z(t(1+\alpha_2^2) - 2\alpha_2 b_2) - k_2^2(1-r\tau^2)^2}\right) \tag{A.5}$$

After getting the reactions of the retailer, the manufacturer maximizes his profit and determines the optimal solutions. The Hessian matrix associated with the profit function Pm is given by

$$H(w, p_d, f_1) = \begin{pmatrix} -2\Gamma_1 & b_1(\Gamma_2 + \Gamma_3) & 0 \\ 2b_1\Gamma_4 & \Gamma_2 + \Gamma_5 & k_1\left(1-r\tau^2\right) \\ 0 & k_1\left(1-r\tau^2\right) & -z \end{pmatrix}$$

In order to prove the programmability of Hessian matrices, the following formulas need to be satisfied:

$$|H_{1\times1}| = -2\Gamma_1 < 0$$

$$|H_{2\times2}| = -2\Gamma_1(\Gamma_2 + \Gamma_5) - 2b_1^2\Gamma_4(\Gamma_2 + \Gamma_3) > 0$$

$$|H_{3\times3}| = -2z(\Gamma_2 + \Gamma_5) + 2zb_1^2\Gamma_4(\Gamma_2 + \Gamma_3) - 2\Gamma_1 k_1^2\left(1-r\tau^2\right)^2 < 0$$

When $\Gamma_1 > 0$, $|H_{1\times1}| > 0$ holds. When $\Gamma_1(\Gamma_2 + \Gamma_5) + b_1^2\Gamma_4(\Gamma_2 + \Gamma_3) < 0$, $|H_{2\times2}| > 0$ holds. When $-2z(\Gamma_2 + \Gamma_5) + 2zb_1^2\Gamma_4(\Gamma_2 + \Gamma_3) - 2\Gamma_1 k_1^2(1 - r\tau^2)^2 < 0$, $|H_{3\times3}| > 0$ holds.

Where $\Gamma_1 = \left(\frac{k_2^2(1-r\tau^2)^2 + z_1(b_2-t)(\alpha_2+1)}{2z(t(1+\alpha_2^2)-2\alpha_2 b_2)-k_2^2(1-r\tau^2)^2}\left((b_2 - t)(\alpha_2 + 1) + \frac{k_2^2(1-r\tau^2)^2}{z}\right) + \frac{k_2^2(1-r\tau^2)^2}{z}\right)$, $\Gamma_2 = \left(\frac{z(1+\alpha_1\alpha_2)}{2z(t(1+\alpha_2^2)-2\alpha_2 b_2)-k_2^2(1-r\tau^2)^2}\left((b_2 - t)(\alpha_2 + 1) + \frac{k_2^2(1-r\tau^2)^2}{z}\right)\right)$, $\Gamma_3 = (\alpha_1 + 1) - (1 + \alpha_1\alpha_2)\left(\frac{k_2^2(1-r\tau^2)^2 + z_1(b_2-t)(\alpha_2+1)}{2z(t(1+\alpha_2^2)-2\alpha_2 b_2)-k_2^2(1-r\tau^2)^2}\right)$, $\Gamma_4 = \left(\frac{b_1^2 z(\alpha_1+\alpha_2)^2}{2z(t(1+\alpha_2^2)-2\alpha_2 b_2)-k_2^2(1-r\tau^2)^2} + 4\alpha_1 b_2 - 2t(1 + \alpha_1^2)\right)$, $\Gamma_5 = \left(\left(-\frac{(b_1(1+\alpha_1\alpha_2))(k_2^2(1-r\tau^2)^2 + z_1(b_2-t)(\alpha_2+1))}{2z(t(1+\alpha_2^2)-2\alpha_2 b_2)-k_2^2(1-r\tau^2)^2} + b_1(\alpha_1 + 1)\right)\right)$.

Therefore, the profit function of the manufacturer π_m is jointly concave in w, p_d and f_1 provided that $\Gamma_1 > 0$, $\Gamma_1(\Gamma_2 + \Gamma_5) + b_1^2\Gamma_4(\Gamma_2 + \Gamma_3) < 0$, and $-2z(\Gamma_2 + \Gamma_5) + 2zb_1^2\Gamma_4(\Gamma_2 + \Gamma_3) - 2\Gamma_1 k_1^2(1 - r\tau^2)^2 < 0$. Mean-while, the unique optimal solution exists.

Bringing (A.4)-(A.5) into the manufacturer's profit function (17), and let $\frac{\partial \pi_m}{\partial w} = 0$, $\frac{\partial \pi_m}{\partial p_d} = 0$, $\frac{\partial \pi_m}{\partial f_1} = 0$.The optimal solutions of w, p_d and f_1 given in Proposition 3 can be found.

Data availability

Data will be made available on request.

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